

# Causality and dispersion relations and the role of the S-matrix in the ongoing research

To the memory of Jaime Tiomno (1920-2011)

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## Abstract

The adaptation of the Kramers-Kronig dispersion relations to the causal localization structure of QFT led to an important project in particle physics, the only one with a successful closure. The same cannot be said about the subsequent attempts to formulate particle physics as a pure S-matrix project.

The feasibility of a pure S-matrix approach are critically analyzed and their serious shortcomings are highlighted. Whereas the conceptual/mathematical demands of renormalized perturbation theory are modest and misunderstandings could easily be corrected, the correct understanding about the origin of the crossing property requires the use of the mathematical theory of modular localization and its relation to the thermal KMS condition. These new concepts, which combine localization, vacuum polarization and thermal properties under the roof of modular theory, will be explained and their potential use in a new constructive (nonperturbative) approach to QFT will be indicated. The S-matrix still plays a predominant role but, different from Heisenberg's and Mandelstam's proposals, the new project is not a pure S-matrix approach. The S-matrix plays a new role as a "relative modular invariant" ..

# 1 Introduction to the various causality concepts along historical lines

Analytic properties of scattering amplitudes which arise as consequences of causal propagation properties were first studied in the context of classical optics in dielectric media and appeared first under the name *dispersion relations* in the late 20s in the work of Kramers and Kronig<sup>1</sup>. The mathematical basis on which this connection was derived amounted basically to an application of Titchmarsh's theorem: a function (more generally a distribution)  $f(t)$  which is supported on a halfline, is the Fourier transform of a function  $a(\omega)$  which is the boundary value of a function whose analyticity domain is the upper half-plane. With appropriate restrictions on the increase at infinity, this analytic behavior can be recast into the form of a dispersion relation (relating the absorptive to the dispersive part) which is the best form for experimental checks of causality. In the simplest case such a relation is of the form

$$a(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Im a(\omega') d\omega'}{\omega - \omega'}, \quad Im \omega > 0$$

Only after world war II this idea of relating spacetime causality with analyticity in the form of dispersion relations found its way into quantum theory (QT). Schützer and Tiomno [3] were among the first who worked out conditions under which a dispersion relation can be derived for scattering amplitudes of elementary processes in the setting of quantum mechanic (QM); later there appeared other contributions with a different adaptation of the notion of causality and slightly different restrictions on the two-particle interaction potentials. These considerations can in principle be extended to a more recent relativistic generalization of QM called "*Direct Particle Interaction*" (DPI) [4][5], a theory which is solely build on particles without using fields or algebras of local observables. In such a setting, in which the Poincaré group is unitarily represented and the S-matrix comes out to be Poincaré-invariant, there is no implementation of micro-causality, similar to the work of Schützer and Tiomno one can only implement macro-causality<sup>2</sup> which includes the spacelike cluster-factorization and Stueckelberg's *causal rescattering* requirements [6][5]. The additional difficulty in the DPI case, which was solved in [4], is that the naive addition of pair potentials in nonrelativistic QM would be in contradiction with the multiparticle representation of the Poincaré group representation and those macro-causality requirements, i.e. those coarse causality requirements which can solely be formulated in terms particles.

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<sup>1</sup>Here and in the following we refer references to the bibliography in [1] wherever it is possible. This monography is a competent and scholarly written account of the subject, though it does not contain the QFT derivation which is based on the Jost-Lehmann-Dyson representation, the latter can be found in [2].

<sup>2</sup>Macro-causality is based on Born-localization and refers primarily to wave functions and their large time propagation. It leads to the concept of velocity of sound in QM and to the velocity  $v < c$  for relativistic mechanics (DPI). Micro-causality is an algebraic property of local observables which entails spacelike Einstein causality and the (timelike) causal completion property.

The main problem in passing from classical optics to QM is that the latter has no finite limiting velocity and admits no wave fronts. Wave packets dissipate instantaneously in such a situation; a finite velocity (as the speed of sound) in a quantum mechanical medium (idealized e.g. as a lattice of oscillators) arises only as an "effective" velocity of a disturbance; more precisely as an asymptotically defined (large time) mean value in wave packet states. In that case localization is related to the spectral theory of the *position operator*  $\mathbf{x}_{op}$  and described in terms of a wave functions  $\psi(\mathbf{x})$  which is a square integrable function on its spectrum. The position operator is frame-dependent (non-covariant) and enters particle physics together with the Born-probability; although the probabilistic "Born-localization" is an indispensable concept of particle physics, it is not intrinsic to QM and has been (and still is) the point of interpretational and philosophical contentions<sup>3</sup>. There is no way to talk about localized observables in QM; the obvious attempt to go to the second quantized Schrödinger formalism and define  $\psi_{op}(\mathbf{x})$  and its local functions as pointlike local generating fields, or to introduce region-affiliated observables by smearing, does not work. These objects loose this property immediately since; unlike for wave function propagation, there is no stable meaning of "effective" on the level of localized observables; quantum mechanical algebras loose their localization at a fixed time instantaneously.

In "direct particle interactions" (DPI) which implements interacting particles in the setting of a multi-particle Poincare representation theory the impossibility of reconciling the particle concept with covariance remains even though with careful implementation of the cluster factorization one can at least obtain a Poincaré covariant scattering matrix with the correct macro-causality properties [5]. The problems of lack of covariance of particle localization and dissipation of wave packets continue in QFT; the probabilistic setting places the particle physics interpretation of the velocity of light on the same conceptual level as that of the velocity of sound, both are "effective" or asymptotic since there is no limiting microscopic velocity.

In QFT one abandons attempts to implement properties of relativistic localization with the help of particles; instead one adapts the idea of a causal propagation of classical wave equations to the setting of QT and postpones the relation with relativistic particles to a second pass. The algebraic side of QFT comes with a very precise definition of causal locality in terms of spacetime-indexed operator algebras  $\mathcal{A}(\mathcal{O})$ ; it consists of two parts

$$\begin{aligned} [A, B] &= 0, \quad A \in \mathcal{A}(\mathcal{O}), \quad B \in \mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})', \quad \text{Einstein causality} \\ \mathcal{A}(\mathcal{O}) &= \mathcal{A}(\mathcal{O}''), \quad \text{causal shadow property, } \mathcal{O}'' \text{ causal completion of } \mathcal{O} \end{aligned} \quad (1)$$

Here the first requirement is the algebraic formulation of the statistical independence of spacelike separated observables; the upper dash on the spacetime

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<sup>3</sup>The most prominent opponent against Born's introduction of the probabilistic aspect to QM was Einstein, even though the resolution of the thermal aspect of the "Einstein-Jordan conundrum" which, similar to the Unruh Gedankenexperiment brought thermodynamic probability into zero temperature QFT [7] may have softened his resistance.

region  $\mathcal{O}'$  denotes the spacelike disjoint region to  $\mathcal{O}$ , whereas on the algebra it stands for the commutant algebra. The second line is the local version of the "time-slice" property [11] where the double causal disjoint  $\mathcal{O}''$  is the causal completion (causal shadow) of  $\mathcal{O}$  i.e. the area of total dependence of  $\mathcal{O}$  which is the algebraic quantum counterpart of classical hyperbolic propagation. The algebras  $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O} \subset R^4}$  form a local net of algebras whose inductive limit is the global algebra. The two causality requirements are not independent. If the Einstein causality can be strengthened to Haag duality<sup>4</sup>

$$\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})', \text{ Haag duality}$$

the causal shadow property would follow. In physically acceptable models observable algebras violate Haag duality only for multiply connected regions; a prominent illustration is given by the QFT Aharonov-Bohm effect [9].

Physically undesirable reasons why QFTs could violate the causal shadow property may in particular occur in correspondences between models in different spacetime dimensions. In that case the lower dimensional contains all the degrees of freedom of the higher dimensional one which is way too much and leads to a violation of the causal shadow property:  $\mathcal{O}''$  contains more degrees of freedom than  $\mathcal{O}$  so that one would have the impression that additional degrees of freedom would have from outside our living spacetime in a mysterious "poltergeist" manner. This cannot happen in the setting of Lagrangian quantization; but since the latter has remained outside mathematical control and only describes a small subset of QFT anyhow (last section), it is indispensable to have a rigorous structural definition outside a Lagrangian setting. The time slice principle [11] was introduced to exclude such unphysical aspects as those of certain generalized free fields.

The causal shadow property does not lead directly to restriction on single operators, but is expected to play a prominent role in securing a complete particle interpretation of a QFT [12][25]. Whereas properties of particles reveal nothing about the more foundational properties of fields and local observables, the latter should lead to the former. We will return to this problem later on.

In its global form, namely for  $\mathcal{O} = \mathbb{R}^3$  at fixed time or a time-slice  $0 < t < \varepsilon$ , it has also been called *primitive causality*<sup>5</sup>. Whereas in this global form it exists also in QM, the local (causal shadow) form is specific for QFT. Without the particle concepts, field variables and the localized algebras which they generate would lead to a mathematical "glass bead game" without much physical content; with the exception of the electromagnetic field a quantum field is not directly measurable, the intervention of particles is indispensable. It is of paramount importance for the physical interpretation of QFT that the two notions of localization, the particle-based *Born-Newton-Wigner localization* of wave functions and the algebraic notion of *causal localization* of local observables, which for

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<sup>4</sup>The exceptional cases where Haag duality for local observables breaks down are of special interest. For the local algebras generated by the free Maxwell field this happens for  $\mathcal{O}$  = toroidal spacetime region (the full QFT version of the semiclassical Aharonov Bohm effect [8]).

<sup>5</sup>The meaning of primitive causality in [1] is slightly different.

finite times behave in an antagonistic way, come harmoniously together in the timelike asymptotic region of scattering. This implies in particular that the *frame-dependent BNW- localization* becomes *asymptotically frame-independent*. Without this asymptotic coexistence there would be no scattering probabilities and hence no particle physics as we know it. Therefore instead of highlighting the misfit of the particle localization concept with that of fields in QFT at finite spacetimes, it is better to emphasize the asymptotic particle/field harmony which is after all everything one needs (the half glass full against the half glass empty view). It is remarkable that in  $d=1+3$  interacting QFT not even a non-compact spacetime localization region as large as a wedge permits the existence of vacuum polarization free one-particle generators (PFG) with a reasonable behavior under translations. Nevertheless in the last section we will present a new approach which extends the intrinsic representation theoretic approach to free particles and fields by Wigner to the realm of interacting QFT; the new idea is: *emulation* of wedge-localized Wick-products of the incoming free field inside the wedge-localized algebra  $\mathcal{B}(W)$  generated by the interacting fields. A special case, which is only available in  $d=1+1$ , leads to the rich class of factorizing models<sup>6</sup>.

The absence of the position operator among the observables of QFT also implies that there is no conceptual basis for the derivation of Heisenberg's uncertainty relation. The thermal KMS properties of the global vacuum state after restricting it to the localized subalgebra  $\mathcal{B}(\mathcal{O})$  offer however an algebraic substitute (see last section): the increase of localization entropy/energy by passing from a "fuzzy" causal horizon (with "attenuation size"  $\varepsilon$  for the vacuum polarization cloud) to the diverging limit  $\varepsilon \rightarrow 0$  (corresponding to infinite uncertainty in QM). Interestingly the restriction of the vacuum to a state resulting from the global vacuum also imports an intrinsic statistical mechanics probability aspect into QFT which is independent from that by Born which entered via scattering theory.

In addition to the above spacelike commutativity and causal completion property, the spacetime-indexed local algebras fulfill a set of obvious consistency properties which result from the action of Poincaré transformations on the localization regions. This is automatically fulfilled if these spacetime indexed operator algebras are generated by finite-component Poincaré covariant fields  $\Psi^{(A,\dot{B})}$  fields (where our notation refers to the well-known dotted/undotted spinorial formalism).

As a result of the "ultraviolet crisis"<sup>7</sup> of QFT, which started in the 30s and lasted up to the beginnings of renormalization theory at the end of the 40s, local QFT became discredited and the introduction of an elementary length into QFT, or its total abandonment giving up localization in favor of an ultraviolet-finite unitary S-matrix, were seriously contemplated. Although QFT enjoyed a strong return, the longings for a S-matrix theory or a nonlocal (often by inferring

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<sup>6</sup>Even in those few cases where they have a Lagrangian name, their existence and the construction of their formfactors cannot be achieved by Lagrangian quantizations.

<sup>7</sup>It was really a crisis resulting from insufficient understandings about QFT.

nonlocality via "non-commutativity") never disappeared even though all these attempts during more than half a century remained unsuccessful (section 3.1)

With doubts about the validity of the principle of causal locality, and in spite of the observational success of perturbative QED in which these principle is realized, there was a strong desire to directly check the validity of the forward dispersion relation, which was done in 1967 [13] for  $\pi - N$  scattering at up to that time available energies. This was the first (and the last) time a model-independent fundamental principle of a relativistic QT was subjected to a direct experimental test in the presence of strong interaction where perturbation theory<sup>8</sup> was not applicable. The importance of this successful test results from the rigorous and profound mathematical-conceptual work which connected the causality principle to the dispersion relation; for an unproven conjecture such a concerted effort could hardly have been justified. Therefore the precise and detailed mathematical work was a valuable investment in particle physics. Whereas in the early work of quantum mechanical derivations of dispersion relations the latter were not the consequence of a principle but rather properties of the chosen potentials, the much more subtle derivation in QFT revealed that they are a direct manifestation of its foundational causal locality principle. The link between this principle and its analytic consequences was a spectral representation for the particle matrix element of two fields, the so called Jost-Lehmann-Dyson representation [2] which generalizes the simpler Källén-Lehmann representation for the two-point function to particle matrix elements of commutators of two fields. It used to be well-known in the 60s and it would go beyond the purpose of this paper to review important results of the past which were lost in the maelstrom of time; for readers who have problems to get the relevant information from the internet, I suggest to visit a library. As a side remark, the JLD spectral representation has been almost exclusively used for the derivation of analytic S-matrix properties; I only know of one quite different use: the Ezawa-Swieca proof [15] of the Goldstone's conjecture based on the generalization of a property of a concrete Lagrangian model stating that a conserved current yields a divergent global charge (spontaneous symmetry breaking) only if a zero mass particle (the Goldstone boson) prevents the large distance convergence<sup>9</sup>.

The formulation of the dispersion relations and their experimental verification was a remarkable achievement in several respects. Besides the aforementioned aspect of a confrontation of a principle directly with an experiment without the intercession of a model, it is the only case of a "mission accomplished" achievement in high energy physics. This is because after the problem had been formulated within the setting of QFT and worked out theoretically, it underwent a successful observational test; the physicists who participated in this unique endeavour could afterwards turn their interest to other problems with

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<sup>8</sup>A perturbative account of dispersion relations and momentum transfer analyticity was presented in [14]. At that time the divergence of the perturbative series was only a suspicion, but meanwhile it is a fact.

<sup>9</sup>The other alternative, namely that the global charge vanishes [16] is related to the Schwinger-Higgs mechanism of charge screening.

the assurance and feeling of satisfaction of having contributed to the closure of one important problem.

It is important not to keep the dispersion relation project separate from later attempts to base particle theory solely on the construction of an S-matrix. The first such attempt, based on a postulated two variable spectral representation of the elastic scattering amplitude was proposed by Mandelstam in 1957 [17]. I remember from my attending seminars at the University of Hamburg that Lehmann Jost and Källén were somewhat unhappy about the increasing popularity of unproven representations. But the main purpose of Mandelstam's representation was not to give an alternative support for dispersion theory but rather to test the possibility of an S-matrix approach to particle theory. In the present work we will not only explain how a foundational understanding of causality leads to a new view of QFT (including the origin of the important crossing property in particle physics), but we will also convince the reader that a lot can be learned from an in-depth understanding of the derailment of all pure S-matrix attempts. In doing this we will learn that the S-matrix has a property which connects it with "wedge-localization" and it is this recently discovered property which makes it an indispensable constructive tool of QFT. In this sense, and only in this sense (and not in the dual model/string theory attempts) Mandelstam's idea about the central position of the S-matrix in particle theory survives.

Naturally a successful concluded project as the unravelling of the connection between causality and dispersion relation invites to look out for extensions into other directions. There were at least two good reasons for this. One was that the successful perturbative approach in QED could not be expected to work in strong interactions (at that time  $\pi - N$  interactions). The other is more profound on the theoretical side and relates to the at that time growing suspicion that renormalized perturbative series in QFT may always diverge; a suspicion which later on became a disturbing fact, since it meant that the only known way to access Lagrangian interactions did not reveal anything about the mathematical existence. This insight relativizes the success of QED somewhat, because to realize that the only remaining possibility, namely asymptotic convergence for infinitesimally small couplings has no useful mathematical status, is a sobering experience. An experimental comparison with perturbation theory is only fully successful if the theory has a mathematical-conceptual existence status. This deficiency of the presently only calculational access distinguishes QFT from any other area of theoretical physics where sufficiently nontrivial soluble examples were available and non-integrable models permit mathematically controlled approximations. The tacit assumption underlying all quantum field theoretical research is of course that this a temporary shortcoming of our capabilities and not a flaw in our characterization of QFT.

The S-matrix bootstrap project, which was vigorously proposed (notably by Chew), was based on the conjecture that a unique S-matrix (primarily of strong interactions), can be determined on the basis of three principles unitarity, Poincaré invariance and the crossing property, where the last requirement extends the analyticity of the dispersion relations. Viewed in retrospect, it is

not these three requirements which cause raised eyebrows, but rather the idea that one can use them to "bootstrap" one's way into finding *the* S matrix of strong interactions. Such ideas about the existence of a unique particle physics "el Dorado" which can be found by juxtaposing the right concepts have arisen several times in particle physics; they are in fact harbingers of the later millennium theory of everything (TOE). One of their features is usually a highly nonlinear property as the unitarity requirement in the bootstrap case. In such a situation "pedestrian" attempts to join linear requirements with nonlinear ones lead in most cases to an explosive nonlinear batch, to which no solution can be found by computational tinkering.

This negative result is sometimes not the end of the story since it may nourish the hope (as in the similar case of the nonlinear Schwinger-Dyson equations) that the principles only allow one solution which, as a result of its uniqueness is very hard to construct. This is the basis of the belief in the existence of a theory of everything (TOE), whereas in QFT the refinement of physical principles only lead to a refined selection of models. The idea that one can nail down a unique solution by physical principles implemented by a bootstrap mechanism had some prominent supporters in the 60s besides its protagonists Chew, Mandelstam and Stapp; also Dyson initially supported this project. QFT avoids such direct encounters with nonlinear properties by implementing unitarity via the asymptotic convergence of Hermitian fields in the setting of scattering theory.

The invigorated QFT in the form of Yang-Mills gauge theories, and the lack of concrete computational bootstrap problems caused a shift away from the bootstrap project towards QFT; As we know nowadays, there is nothing wrong with those bootstrap postulates, the error lies in the expectation that they can be directly used in this raw form to implement calculations, they are simply too vague (in particular the crossing property, see last section).

The few remaining adherents of an S-matrix approach who did not convert fully to gauge theory, turned to more phenomenological motivated problems of strong interactions as the use of the idea of Regge trajectories; this led them to the class of *dual resonance models* [18], which afterwards culminated in string theory. With discrepancies in the observational scattering results involving high momentum transfer, the phenomenological support evaporated. The whole setting was too sophisticated for being supported by phenomenological applications and there arose the idea to connect the orphaned mathematical formalism with more fundamental physics. An argument based on formal manipulations of functional integrals catapulted the phenomenological dual model formalism and its string theory extension into the observational inaccessible setting of gravity at the Planck scale<sup>10</sup>.

In retrospect it is a surprising that this found the unqualified support of the dual model/string community since at that time the critical tradition in particle theory had not yet completely disappeared and there was no common accepted viewpoint of what to make out of the "dual model formalism" involving

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<sup>10</sup>As cynics commented, this was the safest way to prevent any further (after its failure in strong interactions) disagreement with observations.



properties of Gamma functions which was interpreted as a concretization of Mandelstam's double spectral representation of the elastic scattering amplitude. The two additional steps which show that the crossing property of particle theory was not the same as in Veneziano's dual model are one implemented in the dual model are more recent. The first one was the recognition that the formalism of the dual model is based on a crossing property of the Mellin transform of conformal 4-point functions [19][20]. This will be explained in section 3.1. The true crossing property involving particle states on the other hand follows from the thermal KMS properties of the wedge localized interacting algebra together with that of the corresponding free algebra which shares the same modular group. The derivation is based on deep properties of modular localization and will be explained in section 3.2. The crossing property for the S-matrix is a special case of the formfactor crossing since the transition amplitudes are formfactors of the identity operator (section 3.2).

Since causal localization is not only the most important but also the subtlest of all properties of a relativistic QT it is not surprising that errors were committed in the various attempts of its implementation. What is however difficult to understand why it was not possible to repair them within more than 4 decades. Certain scientific developments cannot be understood within the realm of science since they are of a sociological nature. It is evident that an area of research as particle theory, which is by its very nature quite speculative on its unsecured frontier, needs a strong critical corrective for keeping it on track. At the times of competing schools (Landau, Bogoliubov, Lehmann, Jost, Haag) and independent strong individuals as Pauli, the "Streitkultur" in the old world was keeping the particle theory caravan on track and was able to hold course even against some at the time popular fashions which originated in the new world. All this changed when the competing schools were replaced by globalized communities and instead of a Streitkultur, practiced by renowned particle physicists, there was now the unchallenged suggestive power of the reputable community leaders. To make things worse, the newcomers who enter the monocultures practised within a globalized community learn the computational tools in order to become productive within the community, but have no chance to get to that level of critical knowledge which is required to get out of the derailment. This may very well continue into the post LHC situation. Although this is an interesting and important issue to be pursued, our criticism in this paper will be kept on the conceptual-mathematical track. The lack of observational success of string theory which has been the critical focus in several articles, plays no role here. Rather the mere conceptual existence of a trans-QFT setting in relativistic QT which goes beyond QFT in the sense of incorporating its localization concepts as a limiting case, would by itself be a remarkable discovery, independent of its observational status. QFT is the first completely intrinsic and holistic theory [7] (see later) to which for more than 70 years no alternative has been found; every attempt (including S-matrix theories) to find an consistent alternative has failed.

These critical remarks need to be somewhat qualified by emphasizing the recent constructive progress on QFT for theories with an asymptotically complete

particle interpretation that attributes a crucial role to the S-matrix. Besides its well-known role in scattering theory it carries some information about wedge-localization ( $S_{scat}$  is a relative modular invariant). In fact it is precisely this role which allows to formulate the new construction "doctrine" of starting QFT constructions with wedge-localized generators. This is the reverse of the standard doctrine in QFT which is based on local interactions between pointlike free fields and passes to more global constructions including the scattering amplitudes. The advantage of pointlike objects as fields is that they are in the range of the quantization parallelism of classical field theory. But there is a high prize for this to be paid since the analogy to classical fields ignores the QFT phenomenon of localization-caused vacuum polarization. These vacuum polarizations intuitively speaking tend to enforce "Murphy's law": channels which can couple (according to superselection rule) actually will couple. This makes QFT (in contrast to QM) a fundamental theory, but it also renders it inaccessible to functional analytic (single operator) methods. The important new lesson is that generators of wedge algebras have much simpler vacuum polarization properties than quantized pointlike fields; in fact wedge generators present the best compromise in the somewhat antagonistic quantum particle-field relation.

This philosophy was very successful for a class of models which before were treated with the nonperturbative bootstrap formfactor prescription [27]. Similar to the dispersion relation project, it accomplished all its self-posed aims. But as a result of its more modest ambition and guarded presentation, it remained little known to the majority. For this reason it may be helpful to use the remainder of the introduction to present some details about their fascinating history. These models are the only integrable QFTs<sup>11</sup> and as a result of the structure of their S-matrix which factorizes into products of an elastic two-particle scattering function, they are also called *factorizing models*.

They came into being through the observation that certain quasiclassical aspects, first noted by Dashen, Hasslacher and Neveu on some 2-dimensional QFTs (the most prominent was the Sine-Gordon equation) suggested that the integrability was related to the bootstrap properties of their two-particle elastic S-matrices in particular the nuclear democracy principle of its one-particle structure [26]. The existence of an infinite family contradicted of course the TOE uniqueness which was part of the bootstrap doctrine. Even worse for the bootstrap ideologues, each of the purely elastic two-dimensional S-matrices was associated to the system of formfactors of a unique (in an appropriate sense) QFT [27].

The bootstrap-formfactor construction was enriched on the algebraic side by the two brothers Zamolodchikov [30]. I found this new tool quite intriguing and since in my understanding the characterizing concept of QFT is locality, I looked for a spacetime interpretation of these operators and realized that they are better than simply nonlocal, their Fourier transform is wedge-local [31] and this property makes them much more than an mnemonic device; in the form of

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<sup>11</sup>They may be seen as the QFT analogs of the Kepler- or quantum mechanical hydrogen-problem, but whereas integrable systems in the classical or quantum mechanical setting exist in any spacetime dimension, integrability in QFT does not extend beyond  $d=1+1$  [43].

vacuum-polarization-free generators [32] they become a new constructive tool in the setting of algebraic QFT. With their help it was possible to show the true (nonperturbative) conceptual-mathematical existence of interacting factorizing models [28] [29]. After 80 years absence of mathematical control here was a class of two-dimensional models with noncanonical short distance behavior for which all doubts about their existence could be removed and many of the objects one is interested in were computed. The computational difficulties turn out to be opposite to those of perturbation theory, namely the more one moves off mass-shell the more extensive the calculations become: the S-matrix and the formfactors require less computational work, whereas the correlation functions remain prohibitively complicated. Fortunately the existence proof does not require such explicit calculations and neither does the extraction of physical properties as formfactors require the knowledge of correlation functions of fields. For the first time the Lagrangian quantization and functional integral approach in which a less fundamental (classical) theory is supposed to give the tune according to which a fundamental QFT has to dance has been turned around; for most of the models a Lagrangian is not known and there are qualified doubts that a Lagrangian "baptism" exists<sup>12</sup>. This is certainly a respectable success after many decades of stagnation, even if the limitation to  $d=1+1$  still reminds us of the enormous work ahead which will be necessary in order to be able address these problems in realistic  $d=1+3$  models.

These constructions of low dimensional QFTs have led to renewed interest in the origin of the crossing property which plays a crucial role in S-matrix and the more general formfactor properties. Some of the ideas, especially those about the conceptual origin of the crossing property of formfactors, combined with the progress in local quantum physics (LQP), have led to a surprising connection between crossing and the thermal aspects of modular localization. This gave rise to a new setting of QFT in which the S-matrix plays a new constructive role within QFT. It turned out that the analytic crossing identity is related in a deep way to the KMS identity which expresses the thermal aspect of the vacuum restricted to the interacting wedge algebra. Since the KMS property refers to only one algebra one has to "emulate" the wedge localized free fields inside the interacting algebra [20][36]. The aforementioned setting of two-dimensional factorizing models re-appears in this new setting as a special (in fact the only) case in which the emulated operators have simple properties under all translations whereas in the generic case the covariance of the emulated objects does not extend beyond the transformations which map the wedge into itself. In explaining the origin of the particle physics crossing property, the emulation process also unravels an old problem in scattering theory about what happens if the wave packets of incoming particle overlap [34][35]. In this case the threshold singularities do not only limit the validity of the Haag-Ruelle and LSZ scattering theory, but they also lead to radical modification of crossing properties.

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<sup>12</sup>The cardinality of scattering functions obeying the bootstrap principles is bigger than that of renormalizable Lagrangian couplings of free fields.

This new setting could be considered as a heir to the old S-matrix project. The crossing property plays a crucial role in both, but now not as a God-given rule abstracted from Feynman diagrams, but rather as a fundamental consequence of the thermal properties of modular localization. In this way a property from the center of particle physics as crossing is conceptually united with the Unruh effect<sup>13</sup> and with the recent complete understanding of the "Einstein-Jordan conundrum" [7] at the cradle of QFT. This also adds a philosophical touch to these new ways of looking at particle physics.

The new use of the S-matrix however destroy most of the old dreams about the existence of a unique S-matrix theory which can be "bootstrapped" from some postulates (the precursor of the later idea of a TOE). However it does render the S-matrix and particle states important concepts to be used right at the beginning of constructions of QFT, which goes somewhat (but not completely) against Heisenberg's comment "the S-matrix is the roof of the theory and not its foundation" [24] with which he distanced himself from his 1946 S-matrix proposal<sup>14</sup>. This is very different from the present way of dealing with QFTs (e.g. perturbation theory) in which the vacuum correlations of point-like fields of the canonical Lagrangian- or functional- quantization are the main computational tools and only afterwards their large time asymptotic limits or their momentum space mass shell projection are constructed. It should be seen as the heir of the famous "Causality-Dispersion relation project" of the 60s which, although studying on-shell objects, never considered itself a pure S-matrix project.

## 2 Macro- and Micro-causality

The first S-matrix proposal for the construction of relativistic QTs in 1943/46 by Heisenberg [37] was motivated by the desire to overcome the reputed ultra-violet problem as well as to avoid the conceptional difficulties of introducing a short distance behavior improving elementary length into QFT; in a pure global S-matrix setting one would have gotten rid of the two problems. Heisenberg's idea was that one may find sufficiently many properties of  $S$  directly i.e. without having to "interpolate" the incoming and the outgoing particles in a scattering process by interacting (off-mass-shell) pointlike local fields. There was no problem to account for the obvious properties as unitarity, Poincaré invariance and the cluster factorization for large spacelike separation

$$S = e^{i\eta}, \text{ e.g. } \eta = \int \eta(x_1, \dots, x_4) : A_{in}(x_1) \dots A_{in}(x_4) : d^4x_1 \dots d^4x_4, \quad \eta(\dots) \text{ con.} \quad (2)$$

$$\curvearrowright \lim_{a \rightarrow \infty} S(g_1 \dots g_{k+1}^a \dots g_n^a; f_1 \dots f_{l+1}^a \dots f_m^a) = S(g_1 \dots g_k; f_1 \dots f_l) S(g_{k+1} \dots g_n; f_{l+1} \dots f_m)$$

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<sup>13</sup>Also the thermal Hawking effect is localization-caused, in this case the localization boundary is defined in terms of the curved spacetime metric.

<sup>14</sup>Only in factorizing theories their purely elastic S-matrices (only vacuum polarization no on-shell particle creation through scattering) can be computed through the bootstrap project. In higher dimensions there are no theories with only elastic scattering (Áks theorem), real particle creation and vacuum polarization go together and prevent a bootstrap construction.

Here a function  $\eta(x_1 \dots x_n)$  is called connected (*con.*) if it vanishes in the limit of large relative spacelike separation of  $x$ 's;  $f, g$  are wave functions and the upper  $a$  denotes translation by  $a$ . The cluster property for  $S$  (second line) is an immediate consequence of the connectedness of  $\eta$ . Unitarity is trivially satisfied by writing the S-matrix in form of a Hermitian phase operator  $\eta$  and the operational Poincaré invariance follows from that of the coefficient functions  $\eta$  (in general an infinite series), and the cluster property is a consequence of the connectedness of the  $\eta$ 's. But as Stueckelberg pointed out some years later [6], such an Ansatz lacks the macro-causality property which he identified with an S-matrix property called "causal rescattering" [5]. Unlike the three previous properties this property has not and probably cannot be implemented "by hand"; it is automatically fulfilled in models of QFT which have a complete particle interpretation and it also can be implemented in the quantum mechanical direct DPI models [5] mentioned in the sequel. In its simplest version it states that the  $3 \times 3$  S-matrix should contain a particle pole contribution which corresponds to a two-step process: first two of the particles interact and then one of the outgoing particles interacts with the (up this point) noninteracting third incoming particle.

That the second process happens an infinite time *afterwards* means that there is (in the S-matrix idealization) a pole term corresponding to the timelike connection between the two 2-particle processes which has the same  $i\varepsilon$  prescription as the Feynman propagator; the only distinction is that in the present case the latter has only asymptotic validity (in momentum space near the pole). This "causal re-scattering" is an additional requirement on S introduced by Stueckelberg which apparently cannot be implemented "by hand" while maintaining unitarity<sup>15</sup>. This shows that a pure S-matrix theory without using a field-like mediator between incoming and outgoing scattering states is not a realistic goal, a conclusion that also Heisenberg reached some years later [24]. But the first S-matrix attempt was not totally in vain, because by Stueckelberg's suggestive ad hoc simplification of using the Feynman propagator also outside the timelike asymptotic region, and assuming that the interaction region can be shrunk to a point, he independently obtained the Feynman rules through overidealizing macro-causality. So if Feynman would not have found an operational setting for their derivation, we would not have been left completely empty-handed since there would have been a perturbative suggestion by Stueckelberg; however to prove perturbative on-shell unitarity without an operational formalism is not an enterprise whose successful accomplishment is guaranteed. According to an article by Wanders (in [6]) Stückelberg actually found an iterative causal unitarization, starting with a Hermitian Wick-product of fields and invoking micro-causality in every iterative order to restrict the freedom to the structure of counterterms. "Unitarization" by itself is not a well-defined procedure. At the moment one invokes microcausality one has left the realm of a pure S-matrix theory. The perturbative S-matrices one obtains this way are therefore identi-

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<sup>15</sup>It is automatically fulfilled (the causal one-particle structure) in microcausal QFT and can be implemented in the appropriately formulated (see below) DPI setting of relativistic QM.

cal to those from a QFT, in fact the Epstein-Glaser perturbation theory is the mathematically polished form of the Stueckelberg causal unitarization.

Macro-causal structures in scattering amplitudes (without their micro-causal counterparts) are automatically fulfilled in theories with a spacetime dynamics e.g. a Hamiltonian or an equation of motion. The "primitive causality" in Nussenzweig's presentation of nonrelativistic scattering problems [1] is based on the same physical idea, except that (unlike causal re-scattering) one cannot remain within a pure S-matrix setting; the definitions of Schützer and Tiomno [3] use the interaction dynamics for all times. Such macro-causality concepts are quite efficient if one wants to show that ad hoc (not covered by principles) proposals of "modifications by hand", as e.g. the introduction of the Lee-Wick complex poles into the Feynman rules, lead to time precursors and in this way violate primitive causality [38].

These causality properties can be *formulated* in terms of particles, but can they also be computational *implemented* in a pure particle setting i.e. in a dynamics which is formulated *only in terms of particles*? There exists a little known quantum mechanical relativistic multiparticle scheme<sup>16</sup> which leads to interacting multi-particle representations of the Poincaré group and fulfills all the macro-causality properties which one can formulate in terms of interactions between particles only: the setting of *direct particle interaction* (DPI) [4]. Assuming for simplicity identical scalar Bosons, invariant energy operator in the center of mass (c.m.) of two identical particles is  $2\sqrt{p^2 + m^2}$  and the interaction is introduced by adding an interaction term  $v$

$$M = 2\sqrt{\vec{p}^2 + m^2} + v, \quad H = \sqrt{\vec{P}^2 + M^2}, \quad P = p_1 + p_2, \quad p = \frac{(p_1 - p_2)_{c.m.}}{2} \quad (3)$$

where the invariant potential  $v$  depends on the relative c. m. variables  $p, q$  in an invariant manner i.e. such that  $M$  commutes with the Poincaré generators of the 2-particle system which is a tensor product of two one-particle systems.

One may follow Bakamijan and Thomas (BT) [39] and choose the Poincaré generators in a way so that the interaction only appears explicitly in the Hamiltonian. Denoting the interaction-free generators by a subscript 0, one arrives at the following system of two-particle generators

$$\begin{aligned} \vec{K} &= \frac{1}{2}(\vec{X}_0 H + H \vec{X}_0) - \vec{J} \times \vec{P}_0 (M + H)^{-1} \\ \vec{J} &= \vec{J}_0 - \vec{X}_0 \times \vec{P}_0, \end{aligned} \quad (4)$$

where the two particle operators  $\vec{X}_0, \vec{P}_0, \vec{J}_0$  with the subscript zero are just the sum of the corresponding one-particle operators. The interaction  $v$  may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum  $p$  in the c. m. system; but since the scheme anyhow does

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<sup>16</sup>Although Dirac introduced important concepts based on his project of a relativistic particle theory his implementation of a particle-hole theory led to inconsistencies in perturbative orders in which vacuum polarization entered.

not lead to local differential equations, there is not much to be gained from such a choice. The Wigner canonical spin  $\vec{J}_0$  commutes with  $\vec{P} = \vec{P}_0$  and  $\vec{X} = \vec{X}_0$  and is related to the Pauli-Lubanski vector  $W_\mu = \varepsilon_{\mu\nu\kappa\lambda} P^\nu M^{\kappa\lambda}$ .

As in the nonrelativistic setting, short ranged interactions  $v$  lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian [4]

$$\Omega_\pm(H, H_0) = \lim_{t \rightarrow \pm\infty} e^{iHt} e^{-H_0 t} \quad (5)$$

$$\Omega_\pm(M, M_0) = \Omega_\pm(H, H_0) \quad (6)$$

$$S = \Omega_+^* \Omega_-$$

The identity in the second line is the consequence of a theorem which says that the limit is not affected if instead of  $M$  one takes a positive function of  $M$  (6) as  $H(M)$ , as long as  $H_0$  is the same function of  $M_0$ . This insures the asymptotic *frame-independence* (P-invariance) of asymptotic objects as the Møller operators and the S-matrix, but not that of semi asymptotic operators as formfactors of local operators between *ket* in and *bra* out particle states. Apart from this *identity for operators and their positive functions* (6), which seems to play no role in the nonrelativistic scattering, the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property is the statement that  $\Omega_\pm^{(2)} \rightarrow \mathbf{1}$ ,  $S^{(2)} \rightarrow \mathbf{1}$ , i.e. the scattering formalism is identical. In particular the two particle cluster property, which says that for short range interactions the S-matrix approaches the identity holds also for the relativistic case if one separates the center of the wave packets of the two incoming particles. Having a representation theory of the two-particle Poincaré group does not imply that there are covariant local observables, but together with the short range requirement they secure at least the existence of a unitary Poincaré invariant two particle S-matrix which obeys all macro-causality properties in terms of particles.

There is no problem in finding restrictions on the interaction  $v$  which correspond to those which e.g. Schützer and Tiomno [3] used in the nonrelativistic setting. It is however nontrivial to generalize this setting to *multiparticle* interactions since the representation theory of the Poincaré group prohibits a trivial implementation of cluster factorization by adding up two-particle interactions as in the nonrelativistic case. The Coester-Polyzou formulation of DPI shows that this is nevertheless possible [4]. The proof is inductive and passes the clustering of the n-particle S-matrix to that of the n-particle Poincaré group representation which then in turn leads to the clustering of the (n+1)-particle S-matrix etc. There always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix*. Such transformations, which are unfortunately not unique, are called *scattering equivalences*. They were first introduced into QM by Sokolov [40] and their intuitive content is related to a certain insensitivity of the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is reminiscent of the insensitivity of the S-matrix against local changes in the interpolating field-coordinatizations

in QFT<sup>17</sup> in QFT by e.g. using composites instead of the Lagrangian field. From the construction it is clear that this relativistic DPI has no fundamental significance. Its theoretical value consists in providing counterexamples to incorrect conjectures as e.g. the claim that Poincaré invariance of the S-matrix and cluster factorization requires QFT. Its existence sharpens the recognition of the importance of the causal localization and the depth in the particle-field dichotomy of QFT.

### 3 Analyticity as a starting point for a theory?

The two-fold limitation of perturbation theory resulting on the one hand the divergence of its perturbative series and the ensuing doubts about the status of existence of interacting QFT, and on the other hand its limitation to weak couplings and the resulting problems for the description of nuclear interaction (in those days the  $\pi - N$  interactions) led to a return of the S-matrix idea. In many aspects the new bootstrap ideas went beyond the Heisenberg program and its criticism by Stueckelberg, but some of the old conclusions were forgotten. The nontrivial macro-causality properties as the spacelike clustering and the timelike causal rescattering property do not anymore occur in the bootstrap list. Unimportant or forgotten in the maelstrom of time? It is characteristic of the three historical S-matrix projects (Heisenberg/Stueckelberg, bootstrap, dual model/superstring) that each subsequent one added a new idea, but also ignored some of the older messages. As will be seen in more details, the new post renormalization *crossing property* which was not present in the Heisenberg/Stueckelberg setting was simply abstracted from analytic properties of Feynman graphs, but the lack of understanding its physical root-causes carried the seeds of a conceptual derailment whose far-reaching consequences strongly influenced the present situation.

Crossing is most clearly formulated in terms of formfactors, the crossing for scattering amplitudes is a consequence of the formfactor crossing and the LSZ reduction formula. Our proof in the formfactor setting (see last two sections) is very different in scope and physical concepts from the proof for the elastic scattering amplitude derived in the setting of axiomatic QFT using techniques of functions of several complex variables [41][42]. It contains an element of surprise, since the so-called *crossing identity* turns out to be a somewhat camouflaged KMS identity which results from the restriction of the global vacuum to the wedge localized algebra. That the restriction of the global vacuum to the wedge algebra leads to an impure thermal state which is KMS with respect to the wedge preserving Lorentz boost is of course known from the *Unruh and Hawking thermal manifestations of localization* of quantum matter behind causal- or black hole event- horizons<sup>18</sup>. But the manifestation of the thermal KMS identity

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<sup>17</sup>In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class), or in the setting of AQFT by picking another generator from the same local operator algebra.

<sup>18</sup>Contrary to popular opinion it is not the curvature but rather the localization which



(implying that the  $\mathcal{A}(\mathcal{O})$ -restricted vacuum is impure in a very strong sense<sup>19</sup>) in the form of the particle crossing identity is somewhat unexpected (expressed in the German subtitle of section 3.2). Further illustrations of this point can be found in [7] [47].

Analyticity as a postulate standing next to the other physically motivated requirements was an important part of the Chew-Mandelstam S-matrix program (the postulate of "maximal analyticity"). One does not have to share however Chew's and Stapp's opinion that is by itself a physical principle. Rather it is a consequence of the spectrum property and the causal localization principle of QFT, but sometimes the path from the principle to the analytic properties is subtle and demanding. To obtain an impression of the subtleties one only has to look at the details of the derivation of the high energy physics dispersion relation via the JLD representation from the causal locality principles. The title of this section is a characterization of the beliefs at the time of the Chew-Mandelstam S-matrix setting and the answer to the question mark is: yes it is a powerful tool but only if one is able to trace it back to its conceptual origin.

This section will be subdivided into three subsections. The first, entitled a *cul-de-sac*, critically reviews the post bootstrap S-matrix project which started with a concrete conjecture by Mandelstam about the existence of a double spectral representation for the two variable crossing symmetric scattering amplitude. The dual model resulted from a implementation of a particular crossing property which is however not that of particle physics. There are precisely two notions of crossing, that of crossing of conformal 4-point functions by applying the convergent global operator expansion (established in conformal QFT) to the 3 different pairings of fields. In that case the Mellin transform of this expansion produces a sum over infinitely many poles whose position is defined in terms of the anomalous dimension of the composite fields in the expansion. The correct particle crossing, which will be presented in the third subsection 3.3, has nothing to do with this conformal field crossing.

The bootstrap approach was given up, as a result of its incapacity of producing reasonable calculations (it hardly led to any PhD thesis). Its heir, the dual model, was more concrete and, as a result gave rise to many calculations, especially after it became more phenomenological oriented with the attempted incorporation of Regge poles and their trajectories. There remains of course the interesting question: what would have happened if the discovery of the dual model would have occurred in the clear daylight of the Mellin transformed conformal field crossing instead of tinkering with properties of gamma functions; would the dual model still have been considered as a part of the scattering aspects of particle theory?

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generates the thermal aspect. The event horizon attributes to the localization in front of the Schwarzschild horizon a physical reality whereas the causal horizon of a Rindler wedge has a more fleeting existence.

<sup>19</sup>For sharp localization it does not correspond to a density matrix; only if one passes to a somewhat fuzzy surface the algebra becomes a standard  $B(H)$  algebra and the KMS state passes to a Gibbs density matrix state. We believe that there is a relation to 't Hooft's "brick-wall" construction [48]; further work on this point is required.

The use of the dual model for strong interactions was not only given up as a consequence of disagreements with improved scattering data. Its highly sophisticated mathematics was also conceptually out of tune with its phenomenological aims. In this situation the idea that it could be useful in Planck scale physics as a theory of quantum gravity gained ground.

The ultimate step to string theory resulted from an attempt to lend conceptual physical importance to a mathematically tempting formalism by simply forgetting the failed observational connection and postulating the yet unknown gravitational physics at the Planck scale as its new range of application. Our interest in this paper is limited to its problematic relation with the localization concepts of QT, in particular whether it really has the localization property which string theorist ascribed to it [20].

The subsection 3.2 explains why the vacuum state restricted to a spacetime localization region  $\mathcal{O}$  turns into an impure thermal KMS state with a vacuum-polarization cloud hovering in the vicinity of the causal/event horizon<sup>20</sup> [68] [5]; the most interesting case for testing foundational properties of QFT is  $\mathcal{O} = W$  (the wedge region). It also provides additional insight into what physical string-localization really means and why the objects of string theory are not string-but rather point-localized. It also places big marks of doubts on the string S-matrix proposal resulting from pure prescriptive manipulation of functions which by decree are promoted to be scattering amplitudes. Without being able to formulate these recipes in terms of states and operators they are totally unconvincing. Rules which ignore the lesson learned from Stueckelberg's iterative S-matrix construction by an operational unitarization (see previous section) and which simply replace the worldline interpretation for momentum space particle propagators by tubes (worldsheets) without backing up such graphical analogies by Hilbert space operators fall back behind Stueckelberg's work.

Even if this (after more than 4 decades) would still work by some overlooked magic, there is still the conceptual problem of attributing a meaning to a "stringy" S-matrix; an S-matrix is a *global* object which a priori does not contain any information about spacetime localization. In fact for the only known case of *genuine string-localization* as the best possible localization, namely electrically charged matter fields [8][44], the consequence of the weaker than point-like localization is the (since Bloch and Nordsiek well-known) phenomenon of infrared divergences in scattering theory which leads to the abandonment of the S-matrix in favour of photon-inclusive cross sections (for which unfortunately no elegant LSZ like representation in term of spacetime correlations has yet been found).

The second subsection of section 4 presents a new constructive setting in which an algebraic version of crossing and *analytic exchange* (explained there) is the starting point of a new constructive approach which in  $d=1+1$  results in an existence proofs and the explicit construction of formfactors for the class of factorizing models. In a certain sense this success vindicates at least some

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<sup>20</sup>The localization-induced vacuum polarization may be seen as the metaphor-free aspect of the "broiling vacuum polarization soup" of the books on QFT [?].

aspects of the aspirations of the old dream of the bootstrap community and in particular of Mandelstam [17] concerning the importance of analytic properties of on-shell objects, even though its implementation requires quite different concepts as well as a return to QFT. Nevertheless the use of the S-matrix as a basic computational tool (and not just the roof of local particle physics) is shared with the S-matrix attempts of the 50s and 60s which now, together with formfactors of fields and a new much more subtle role of the crossing property, forms the backbone of the new approach.

### 3.1 Important lessons from a cul-de-sac

The first two attempts to avoid fields in favour of a pure S-matrix approach failed basically for the same reasons. The nonlinear unitarity of the S-matrix together with other physically motivated linear requirements results in a rather unwieldy computational batch. In the eyes of some created the impression that if such a system of requirements admits any solution at all, then it should be rather unique. In this way an early version of a theory of everything (still without gravity) was born. But the disparity between high dreams and the difficulty to translate them into credible computations led to an early end of the projects. Young newcomers who were looking for doable and credible computations were better served by the new nonabelian gauge theories for weak and strong interactions.

The dream of uniqueness of the bootstrap ended at the beginning of the 70s with the little noticed construction of an infinite family of elastic scattering functions in  $d=1+1$  [27] which fulfill the bootstrap requirements. But in contrast to the missionary zeal (if not to say cleansing rage) against QFT by the adherents of the bootstrap project, this modest observation about *factorizing models* showed that each S-matrix came precisely with one QFT whose scattering it describes. This relation strengthened the idea of a unique association of a QFT to an S-matrix (i.e. the uniqueness of the inverse problem of scattering theory in QFT)<sup>21</sup>. Most of the factorizing models, whose construction is governed by S-matrix ideas, have no known Lagrangian. The important scientific legacy of the bootstrap era is the idea that there are construction methods of QFT which are not only outside the Lagrangian and functional quantization methods, but which for the first time are capable to secure the mathematical existence of models of QFTs.

At the time of the bootstrap Mandelstam [17] conjectured a representation of the elastic scattering amplitude which contained the dispersion relation as well as an extension of the at that time known momentum transfer  $t$ -analyticity. In contradistinction to the nonlinear bootstrap program it seemed more susceptible to computational ideas, at least if one left out unitarity. For the project of establishing the observational validity of the (model-independent) causality principle underlying QFT via an experimental check of the dispersion relations,

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<sup>21</sup>An S-matrix does however not distinguish a particular field, rather it associates to a local equivalence class (Borchers class) or more compactly to a unique net of local operator algebras of which those fields are different generators.

this conjectured but never proven representation would have little interest; in this case the rigorously established Jost-Lehmann-Dyson representation clinched the connection between *causality and the dispersion relation project*.

There was a second more phenomenological motivated train of thought involving the idea of analytic continuation in the angular momenta and the ensuing connection of the related Regge trajectories (particles, resonances) which suggested observational correspondences with the real world of strong interactions. In this situation Veneziano, guided by the Mandelstam representation, produced a formula which combined infinitely many particle poles into a trajectory in such a way that the relation between Mandelstam's s and t channels appeared as a implementation of the crossing property of QFT; as a result the model was called the dual model. This was achieved in an ingenious by mathematical properties of Gamma and Beta functions, so that the result appeared to some as a profound confirmation of the underlying phenomenological ideas and to others too mathematical for the phenomenological use.

This construction, which appeared at the beginning to be unique, admitted several similar solutions [46]. As we know nowadays, the correct interpretation of these somewhat magic constructions (they were not derived or related to physical principles) has nothing to do with the world of S-matrices of particle physics. They rather result from the appropriately normalized *Mellin-transforms of correlation functions of conformal covariant fields* [19]. A conformal 4-point function can be globally operator-expanded in 3 different ways

$$\begin{aligned} & \langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4) \rangle \\ & A_i(x)A_l(y) = \sum_k \int F_{i,j}^k(x,y,z)C_k(z)d^4z \end{aligned} \tag{7}$$

The global expansions used in the second line (only) hold in conformal theories; in general one can only establish the asymptotically convergent Wilson short distance expansions. As Mack has shown [19] there is an analogy of conformal QFT with particle physics in which the charges of charge-carrying fields correspond to particle momenta and their anomalous dimensions to mass squares. This analogy is especially treacherous if one thinks of the Mellin poles, which lie on the trajectory of the anomalous dimensions of the composite fields  $\dim C_k$ , as the one-particle poles of the Born approximation of a hypothetical scattering amplitude in a relativistic QT with infinitely many particles. The three ways of operator-expanding the conformal 4-point function correspond formally to the three s,t,u crossed channels. The interpretation as a crossing symmetric Born approximation in the presence of an infinite tower of interacting particles is far-fetched, even phenomenological uses of a mathematical formalism cannot completely ignore its underlying physical principles which point into a quite different direction; it does however explain the bizarre scientific appearance which string theory has even to those who do not command over the conceptual tools used in the present critical evaluation. For more detailed discussion see [47].

One may speculate about what would have happened if this observation would have arisen in this way and not as it did as a result of experimental

mathematics through tinkering with gamma functions which leaves more leeway for phenomenological interpretations. Conformal field theory is *the* field theory par excellence without a particle interpretation<sup>22</sup>; its use is limited to the study of structural problems in QFT. At the time of the invention of the dual model the above conformal construction was not known. But in plain view of the very different origin of the particle physics crossing (see next section) a defence of the dual model/string theory as a description of particle scattering has no basis. As will be seen in the sequel, the same applies to the use in the sense of a two-dimensional sigma model with "target-space" indices referring to higher-dimensional spacetime (the source-target embedding idea)

The mass/spin tower position of the poles in the dual model can also be obtained from the quantization of a Lagrangian model: the bilinearized Nambu-Goto Lagrangian<sup>23</sup> leads to a wave function space which contains a tower of irreducible (m,s) Wigner representation of the Poincaré group as well as oscillator operators which link the levels with increasing (m,s). Its second quantization is the *string field*, an unfortunate name for a dynamic infinite component pointlike localized field since the name has already been used for fields causally localized along an semiinfinite spacelike string (unfortunately this point remained unmentioned in [60]). Here the prefix "dynamic" is important because a direct sum in the wave function or tensor product in the second quantized setting would hardly be worthwhile studying; dynamic stands for the presence of operators which change the composition within the tower and hence the contribution of the individual components to the field<sup>24</sup>. In fact the exponential of the field variables  $X^\mu(\sigma,\tau)$  of the model contrary to the belief of string theorists describe (after splitting off the c.m. motion) a change in the internal composition over a localization point. In a geometric terminology the change is upward and not sideways in the target space; it resembles a change of the spin and not a spread in the pointlike position.

That string theorists have a disturbed relation to causal localization and covariance is obvious from their supporting use of the classical Lagrangian of a pointlike particle [50]

$$\mathcal{L} = \sqrt{ds^2} \curvearrowright x^\mu(\tau), \text{ cov.orbit} \tag{8}$$

*no quantum counterpart*

which is the well-known Lagrangian describing the free movement of a classical particle in an arbitrary  $g_{\mu\nu}$  spacetime metric. This one-dimensional "support" for ST backfires totally since a covariant quantum theory of relativistic particle orbit does not exist; there is simply no frame-independent particle position

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<sup>22</sup>Any canonical conformal free field is a free field and the LSZ asymptotic limit of any anomalous dimensional field vanishes [49] [43]. No inclusive cross section construction as used in QED led to an observable which could be interpreted in terms of particles.

<sup>23</sup>The original Lagrangian containing square roots of quadratic form in  $X^\mu(\sigma,\tau)$  leads to the Pohlmeyer invariants which have no relation to string theory [59].

<sup>24</sup>As any field resulting from Lagrangian quantization, the string field is irreducible (the mathematical meaning of "dynamic") whereas a direct sum is not.

operator a fact which Wigner knew already in 1939 when he proposed to characterize relativistic particles<sup>25</sup> not by (nonexisting) quantum mechanical position variables but rather in terms of classifying the representation spaces of their wave function from which the quantum fields (which do admit a covariant localization) result by second quantization; its non-existence is one of strongest reasons for doing QFT instead of relativistic QM (see a critical evaluation of the "direct particle interaction" [5]). There is no covariant QT which originates from quantization of this Lagrangian and there are many other cases of classical Lagrangians whose formal quantization does not lead to what one incorrectly expects, namely a covariant quantum theory. The similar looking classical Nambu-Goto Lagrangian leads to the same problem; the classical variable has no quantum counterpart  $X_\mu(\tau, \sigma)$  which describes a covariant object in spacetime. Rather what happens with using the (quadratic modification of) this Lagrangian in ST is that (apart from the zero mode which describes the covariant position of a pointlike object) the oscillator degrees of freedom in the Fourier transforms of such objects go into internal degrees of freedom residing in a Hilbert space which has nothing to do with spacetime localization. The string field is a pointlike localized field and the only candidate for a nontrivial *dynamical infinite component field* of the kind which Barut, Kleinert and others tried to construct it (in vain, because they used extensions of the physical L-group) [14]. All correct calculations of "quantum strings" led to pointlike (graded) commutators, there are no "points on a string" as in [56] [55].

Actually such dynamical infinite component fields are very hard to construct and according to my best knowledge the ST construction is the only one. In fact Lagrangians which lead to quantum objects with a noncompact internal symmetry space ("target space") only arise from "second quantization" of non-rational chiral QFTs since the existence of a continuous charge superselection structure is the prerequisite of a noncompact target space. There are simply no higher dimensional quantum field theories with such a property, a noncompact index structure of such quantum fields must always comply with the spacetime dimension in which they live (tensor/spinor indices). A more fundamental theory (QFT) does not have to dance to the tune of a less fundamental one (classical field theory) i.e. the quantization parallelism only works with careful selected input; neither do classical Lagrangians always have quantum counterparts nor do QFT models always have a classical counterpart. In the case at hand: covariant operators  $X_\mu(\sigma, \tau)$  which are associated with a "quantum surface" in spacetime do not exist (they become "internal") and the claim that ST leads to gravity cannot be upheld; string theory is a gigantic distorting mirror for particle physicists albeit a valuable source of intuition for mathematicians.

It should be added that in relativistic QM (DPI) the frame dependence of the particle position operator does not prevent the large time part of wave functions

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<sup>25</sup>Wigner was hoping that his relativistic representation theory would lead to an intrinsic access (without invoking the quantization of classical structures) to QFT. Whereas the hope was well-founded and found its later realization in the notion of modular localization, the frame-dependent Newton-Wigner localization (the Born localization adapted to the relativistic inner product) was not what he had hoped for.

to describe covariant trajectories of their c.m., if this would not be so scattering theory would not even work in QFT. In fact in QFT the non-covariant localization associated with a particle position operator and the covariant localization through fields coalesce for infinite timelike separation of events, and hence the S-matrix matrix is Poincaré invariant [5]. It is this property which in QM allows to talk about the (material-dependent) velocity of sound in the sense of an effective velocity even though the wave packets dissipate.

With the exception in chiral QFT (presented below) inner symmetries of scalar quantum fields, often referred to as sigma-model fields, must transform according to finite representations of compact groups. This is a consequence of the profound DHR analysis of superselection rules combined with the DR construction of field algebras. These theorems use the causal localization of QFT which is deeply related to vacuum polarization and thermal properties and as a result is much more restrictive than the geometrical localization of classical fields. Classical sigma model fields on the other hand may carry noncompact representations of inner symmetries. This has an obvious generalization to arbitrary fields: non-scalar quantum fields can, besides compact internal symmetries only carry the vector/spinor indices which are associated with the spacetime on which they "live" whereas classical fields which live on d-dimensional spacetime (the source space) could carry a  $D \neq d$  "target space" symmetry. Apart from the exception of chiral theories such a source-target embedding is not possible in QFT.

This remarkable exception coming from chiral theories owes its existence to the fact that there exist besides "rational" chiral models which have a finite or countable number of superselection sectors also lesser studied models with a continuous number of inequivalent representation sectors. According to the DHR superselection theory this cannot happen in higher dimensional theories [51]. The simplest such model can be defined in terms of a D-component current [52][53]. Defining potentials  $\Phi_i$  and using the weakly convergent integrals over currents which represent the global charges one has

$$\begin{aligned} \Phi_i(u) &= \int_{-\infty}^u j_i(u') du', \quad i = 1..D, \quad \langle j_i(u), j_k(u') \rangle \sim \frac{\delta_{i,k}}{(u - u' + i\varepsilon)^2} \quad (9) \\ \Psi(u, \vec{q}) &= \exp i \vec{q} \cdot \vec{\Phi}(x), \quad Q_i \Psi(u, \vec{q}) |0\rangle = q_i \Psi(u, \vec{q}) |0\rangle, \quad Q_i = \Phi_i(\infty) \end{aligned}$$

$$\begin{aligned} Q_i &\rightarrow P_i \quad \text{multcomp. charge } q_i \sim \text{particle momenta } p \\ \vec{q}^2 &= \dim \Psi, \quad \text{particle mass } p^2 \sim \text{anomalous dim.} \end{aligned} \quad (10)$$

The last two lines contain an analogy between the charge- and momentum labeling but it needs to be emphasized that at this point this is only suggestive. This model has a rich use in mathematical physics. The general commutation relations between the  $\Psi$  are plektonic (braid group) and one of the interesting problems has been to classify the maximal local extension of the vacuum representation of the  $j_i$  algebra. This can be done in terms of even lattices. Among

those maximal observable algebras there are some which have no superselection sectors and belong to the largest exceptional finite groups (the moonshine group).

The use of this chiral sigma model for particle physics consists in trying to convert the above analogy into a situation where the D-dimensional space carries a representation of the Poincaré group. This requires a Fourier decomposition of the compactified conformal field  $\Phi_i$  and the use of the zero mode as the coordinate  $x_i$  whereas the higher oscillator modes describe the changing internal degrees of freedom and not a movement in target spacetime. With other words the sigma field remains point-like localized<sup>26</sup> since the infinitely many oscillator degrees of freedom build up an internal space over the localization point. In [55] [56] the authors computed the commutator of the infinite component string field and obtained the correct result of its pointlike nature. But (perhaps by being a member of the string community) they overlook this important point and rather create the impression that one is confronting a string of which only one point is visible. Trying to please a community or self-delusion?

If the Poincaré group representation is required to be unitary and with positive energy then the answer is almost unique<sup>27</sup>: it must be the D=10 super string representation respectively one of its finitely many M-theoretic variations. Such a nearly unique answer is always surprising; no representation at all or infinitely many would have attracted less attentions. Nevertheless it is hard to understand the immense physical-philosophical leap from the observation that non-rational chiral sigma models can carry representations of noncompact groups (in particular of the Poincaré group) to the invocation of a fundamental theory of our living spacetime.

Even in this case of the use of non-rational chiral models the representation on the target space of continuous superselection sectors the target localization remains pointlike, there is no embedding of a chiral theory in a higher dimensional target space; in fact a lower dimensional QFT can never be embedded into a higher dimensional target theory, QFT is too holistic<sup>28</sup> in order to allow such a possibility; for an explanation of "holistic" see [7] [47]. A string-like localized object cannot be introduced in terms of embedding; the only known way to obtain a nontrivial<sup>29</sup> string-like localized structure in a higher dimensional spacetime is to introduce an elementary *stringlike localized field*. An example for a covariant stringlike free field is provided by the infinite spin Wigner representations [45]. These fields have no Lagrangians in contrast to the fields of string theory. So

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<sup>26</sup>It is a solution of the old problem of finding dynamical infinite component fields for which prior attempts to generate an representation containing an interesting infinite (m,s) tower spectrum failed [54].

<sup>27</sup>The requirement that a unitary positive energy representation of the Poincaré group acts on the inner symmetry target space of a non-rational chiral sigma model determines a mass/spin spectrum.

<sup>28</sup>Holistic refers to the fact that that localization in QFT is, different from QM since it is always accompanied by thermal manifestations and vacuum polarization at the causal horizon of the spacetime localization region. In [7] and [47] the reader finds a more detailed presentation and illustrations of this concepts.

<sup>29</sup>One which is not representable as a line integral over a pointlike field.



this state of affairs may be paraphrased by: string theory is Lagrangian but not string-localized and string-localized fields are genuinely string-localized but do not arise from Lagrangian quantization. The Nambu-Goto and the Polyakov Lagrangian are classical strings but their canonical quantization do not produce *quantum world sheets* inasmuch as the relativistic particle Lagrangian does not produce a *quantum world line*. If one wants to associate a quantum theory with the N-G Lagrangian it is the theory of Pohlmeyer's invariants [59]. They are the quantum counterparts of classical invariants associated with the N-G Lagrangian, but their quantum interpretation is unknown.

The above critique applies to the embedding interpretation of objects  $X^\mu(\sigma, \tau)$   $\mu = 1..D$  in functional integrals as world sheets in a D-dimensional target space. One has no control over the localization of such objects and it should therefore be no surprise that these degrees of freedom go, apart from a zero mode which goes into the pointlike localization, into the internal degrees of freedom over a point. The holistic structure of QFT forbids any form of embedding. This affects also Polyakov's Lagrangian formulation of string theory and its use as a basis of gravity. It also affects the quantum counterpart of the Kaluza-Klein dimensional reduction. A QFT characterized by correlation functions with an analytic continuation to euclidean points can be dimensionally reduced by "thermalization" in one euclidean coordinate and subsequently taking the infinite temperature limit which shrinks the periodicity  $\beta \rightarrow 0$  and may be interpreted as a "curling up" one coordinate. Granting sufficient analyticity this construction via thermal states maintains the original algebraic structure and remains therefore intrinsic. On the other hand, Kaluza-Klein arguments on the level of the Lagrangians instead of the quantum correlation functions of a model [57] are not trustworthy since they ignore the holistic structure of QFT correlation functions.

A related mechanism to lower the spacetime dimension of a QFT is the holographic correspondence/projection. The best known case is the  $AdS_{d+1} \longleftrightarrow CFT_d$  correspondence which relates models on d+1 dimensional Anti-de-Sitter space with conformal fields. Whereas the descend from the higher to the lower dimensional theory can still be formulated in terms of spacetime limits of AdS fields, the inverse map is more conveniently expressed in an algebraic setting [58] since there is no pointlike correspondence. Although this correspondence can be rigorously established on the mathematical side, it suffers from a serious physical defect which shows up if one asks questions about the cardinality of phase space degrees of freedom. Intuitively one would expect to find an "overpopulation" on  $CFT_d$  if one starts from a "healthy" QFT on  $AdS_{d+1}$  and an "anemic"  $AdS_{d+1}$  theory in the opposite case.

This is indeed the case; although both theories are Einstein-causal, the too many degrees of freedom in the lower dimensional theory lead to a breakdown of the causal shadow property 1. This can be made explicit by computing the result starting with a free massive field on AdS in which case the correspondence leads to a conformal generalized free field, an object which violates this aspect of the causality property and for who's exclusion the causal shadow property was introduced in [11]. The rationale for this at that time was that although it is desirable to dissociate QFT from the quantization parallelism to classical

field theory, one should preserve all those properties which are formally part of the Lagrangian formalism (the causal Cauchy propagation) but which allow a formulation within a relativistic QT. In a later work [12] this was connected with the cardinality of phase-space degrees of freedom which in QFT mildly infinite (compact, nuclear) [25] in contrast to QM where it is finite per cell of phase space.

Another closely related variation of the same theme is the conceptual interpretation of the restriction of conformal QFT to a lower-dimensional "conformal brane". In this case the too many degrees of freedom can be encoded into a huge internal symmetry [19] which however does not change the unphysical aspect. Since D-branes have only been proposed as quasiclassical objects, the issue of degrees of freedom remained unaddressed.

Phase space arguments were not really popular in the old days because QFT outside Lagrangian quantization was not considered a pressing issue; in more recent times, when the degree of freedom issue could have had a moderating influence on an issue (on which part of QFT came apart at its seams in the flurry of the Maldacena conjecture), it was largely forgotten.

Since our conclusions about all problems in which causal localization come into play were negative, let us try to understand whether at least the retraction to a pure S-matrix setting, forgetting Lagrangians and string fields, is consistent. Here it is instructive to take a critical look at the first attempt by Heisenberg and somewhat later by Stueckelberg to formulate particle physics in a pure S-matrix setting. As mentioned in the introduction Stueckelberg added the idea of macrocausality to Heisenberg's Ansatz in terms of an exponential phase operator which only satisfied the cluster-factorization of S. This requirement led him to the asymptotic structure of causal rescattering in terms of particle lines representing Feynman propagators. By extrapolating this structure for all distances and assuming that the interaction region is pointlike, he found the Feynman rules, i.e. by the extension of the macrocausality property for particles beyond their range of validity, he arrived at the perturbative rules of micro-causal QFT.

It is interesting to mention that Stueckelberg's approach [6] actually amounts to an iterative unitarization process for an S-matrix, starting with the first order

$$S = 1 + \sum_{k=1}^{\infty} S_k, \quad S_1 + S_1^* = 0 \quad (11)$$

$$S_2 + S_2^* = -S_1 S_1^*, \quad S_3 + S_3^* = -(S_1 S_2^* + S_2 S_1^*), \text{ etc}$$

He started (in modern terminology) from a anti-Hermitian  $S_1$  in terms of a spacetime integral over a scalar Wick-ordered polynomial in free fields. Since in each order only the Hermitian part is determined by the previous orders, he imposed micro-causality, even though in contrast to macro-causality this had no cogent reason in a particle S-matrix setting and requires to write  $S_1$  in terms of pointlike free fields. He then showed that this restricted unitarization scheme determines S up to local counterterms and the result is then identical to Feynman's time-ordered formalism and therefore imposing the micro-causality

on the operator densities of the various  $S_k$  is equivalent to perturbative QFT<sup>30</sup>.

Forgetting for a moment that the dual model is based on the Mellin transform crossing property rather than the particle physics crossing (next subsection) let us ask the question: what does it mean to obtain the  $S_{scat}$  of string theory by unitarizing the dual model? Where is the first order operator in a Hilbert space which corresponds to the string S-matrix analog of Stueckelberg's  $S_1$ ? And what controls the higher order anti-Hermitian parts? And last not least what does the notion "stringy" with its spacetime appeal mean, assuming that the stringyness of an S-matrix (whatever it means) entered the theory through the dual model. Does it imply reading back an S-matrix property into an imagined off-shell property? Why should calculations of functions called scattering functions by fiat, based on splitting and fusing tube graphs (which for 50 years resisted their presentation in terms of operators and states) have anything to do with QT?

In the next subsection it will be shown that despite all these failures of S-matrix based approach to particle physics the S-matrix remains the most important tool in a non-perturbative approach to particle theory. This new approach will also lead to a derivation of the particle crossing property and lend new strength to the belief that the S-matrix is not only the roof of QFT but also an indispensable tool in the nonperturbative classification and construction of its models.

### 3.2 Modular localization, its thermal manifestation and the origin of particle crossing

IN DIESEM FALL UND UEBERHAUPT, KOMMT ES GANZ ANDERS ALS MAN GLAUBT.  
(W. BUSCH)

Many properties in QFT allow a more profound understanding (beyond the mere descriptive presentation) in a formulation in which one deals with space-time indexed systems of operator algebras rather than with their generating point- or string-like localized fields. An illustration was given before within the algebraic formulation of causality, in which the *causal shadow property* is more natural than its formulation in terms of quantized Cauchy data for individual fields. This is in particular true about the thermal properties which result from the restriction of the vacuum (or other finite energy states) to local subalgebras.

This thermal manifestation of subalgebras is a holistic property par excellence; although the KMS which characterizes this manifestation is shared by all individual operators which share the same causally completed localization

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<sup>30</sup>The iteratively constructed  $S(g)$  is the Stueckelberg-Bogoliubov-Shirkov operator functional which depends on space-time dependent coupling function. The S-matrix (formally for constant  $g$ 's) is related to this functional by the "adiabatic limit" whose existence is roughly equivalent to the asymptotic convergence of fields in the LSZ scattering theory. In many theories (eg. QED) this limit does not exist as a result of the infraparticle phenomenon [9].

region, its conceptual understanding is only possible in terms of ensembles of observables; just as in the global standard "heat bath" statistical mechanics. Whereas in QM such situation arises mainly through a coupling to heat bath within the thermodynamic limit of Gibbs density matrices, in QFT the restriction of the vacuum (or any other finite energy state) to a localized algebra creates a singular impure KMS state without the coupling to an external heat bath.

The appearance of "localization thermality" in QFT (in contrast to the global heat bath caused thermal aspects of thermodynamics) has far-reaching consequences. In a way it vindicates Einstein's steadfast rejection of Born's probability as an attribute of an individual quantum mechanical event. In his dispute with Jordan<sup>31</sup> [61] both of them missed to notice this important conceptual attribute of Jordan's new "wave quantization". Its early recognition could have changed the history of the philosophy of QT (in particular concerning the measurement process) and reconciled Einstein with its probabilistic ensemble structure.

A property which comes directly from the adaptation of the "action at the neighborhood principle" of Faraday and Maxwell to QT [22] to QT and does not have to be added "by hand" (as Born's quantum mechanical probability<sup>32</sup>) would almost certainly have received Einstein's approval. In QFT the basic observables are localized; quantum mechanical global observables and the assignment of probabilities to individual events as a *practically useful idealization* (and not as a foundational property) hardly cause philosophical headaches. It is regrettable that philosophers of science have remained with Schrödinger's cats and Everett's many-world interpretation; the messages coming from the less metaphoric and more fundamental local quantum physics [25] have largely been ignored. In the early days the holistic aspects of QFT as manifested in vacuum polarization and thermal aspects of localization were not understood since QFT was thought of in terms of relativistic QM [5], but there is no reason why even nowadays questions of interpretation of QT are mainly discussed in the setting of the less fundamental QM.

In the following we will present some important results from local quantum physics [25]; the reader who is unfamiliar with these concepts should consult the literature [5].

Spatial separation of a quantum mechanical global algebra into two mutually commuting spatially separated (inside/outside) subalgebras leads to a *tensor factorization* of the quantum mechanical vacuum state and the phenomenon of *entanglement* for general states. Nothing like this happens for local subalgebras in QFT. Rather they radically change their algebraic properties; instead of being

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<sup>31</sup>Nowadays referred to as the Einstein-Jordan conundrum [21]. It played an important role in Heisenberg's later discovery of vacuum-polarization, but the understanding of localization thermality had to wait more than 6 decades.

<sup>32</sup>The localization probability of a Schrödinger wave function and its relation to the spectral decomposition of the localization operator was introduced soon after Born defined the scattering probability (cross section) for the Born approximation. by Pauli; it appears as an added footnote in Born's paper.

equal to the algebra of all bounded operators on a smaller Hilbert  $B(H_{red})$  space referring to the spatially reduced degrees of freedom as in QM, the localized subalgebras of QFT belong to a different type called "hyperfinite type III<sub>1</sub> factor algebra" (in the Murray-von Neumann-Connes classification) for which among several other changes the above tensor-factorization breaks down. For reasons which will become clear later on, we will call this operator algebra type shortly a *monad*, so every localized algebra of QFT is a *monad* [5]. Its occurrence in QFT is inseparably related to the vacuum polarization and thermal properties of localization in QFT.

Although the division into a spacetime region and its causal complement leads by use of Einstein causality to the mutual commutativity, the *vacuum does not factorize* and hence the prerequisites for the usual form of entanglement are not fulfilled; this is in spite of the fact that the algebra associated to a spacetime region and its commutant together generate the full global algebra  $B(H)$ . In fact a *monad has no pure states* at all, rather all states are *impure in a very singular way*, i.e. they are not density matrix states as impure states in QM; the vacuum restricted to a local monad turns into a singular KMS state. Such states appear in QM only in the thermodynamic limit of Gibbs states on box-quantized operator algebra systems in a volume  $V$ . Later we will turn to the mathematics behind these observations which is *modular operator theory* and more specific *modular localization*.

Usually people are not interested<sup>33</sup> in an intrinsic description of the thermodynamic limit state (called "statistical mechanics of open systems"), but if they were, they would find, as Haag, Hugenholtz and Winnink in 1967 [25], that the limiting state ceases to be a density matrix state and becomes instead a singular KMS state i.e. a state which has lost its trace property and hence its Gibbs representation property (volume divergence of partition function). Instead it fulfills an analytic relation which first appeared as a computational trick (to avoid computing traces) in the work of Kubo, Martin and Schwinger and later took on its more fundamental significance [25] which it enjoys presently as one of the most impressive links between physics and mathematics. Whereas monads with singular KMS states appear in QM *only* in the thermodynamic limit of finite temperature Gibbs states, their occurrence in QFT is abundant since any reduction of the vacuum onto a causally closed localized subregion leads to such an impure state which is KMS "thermal" with respect to the modular Hamiltonian on a monad (see below).

With a "split" of size  $\varepsilon$  between the subalgebra and its causal complement<sup>34</sup>, one returns to a situation which resembles QM in that the global algebra tensor-factorizes. The possibility of doing this is called "the split property" [62] and

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<sup>33</sup>For example the tensor factorization formalism known as "thermo-field formalism" breaks down in the thermodynamic limit for the same reasons as the Gibbs density matrix description, i.e. this formalism is not suited to describe "open systems".

<sup>34</sup>The possibility of doing this is called "the split property" [62]. Whereas the standard box quantization does not allow to view the boxed system as a subsystem of a system in a larger spacetime, the splitting achieves precisely this at the price of vacuum polarization at the boundary. The physics based on splitting is called "open system" setting.

it is closely related to issue of what replaces the quantum mechanical finite phase space degrees of freedom per unit phase space cell in QFT [25] and the closely related question of the causal propagation as expressed in the time-slice property and "Haag duality". Whereas the standard quantum mechanical box quantization does not allow to view the boxed system as a subsystem of a system in a larger spacetime, the split property achieves precisely this at the price of vacuum polarization at the boundary. The physics based on splitting is called the "open system" setting [25].

The state in which the system "splits" is a highly entangled Gibbs density matrix state with respect to a "split Hamiltonian" which approximates for  $\varepsilon \rightarrow 0$  the modular Hamiltonian (see below) which is determined by the localized algebra and the vacuum state  $(\mathcal{A}(\mathcal{O}), \Omega)$ . There are presently two pictures leading to two formulas which are different by a logarithmic factor: the lightlike sheet picture and the analogy with Heisenberg's partial charge behavior. The latter leads to (n=spacetime dimension)

$$entropy \sim \begin{cases} \ln(\frac{R}{\Delta R}), & n = 2 \\ (\frac{R}{\Delta R})^{n-2}, & n > 2 \end{cases} \quad (12)$$

where the region is a ball of radius  $R$  and  $\Delta R$  the "fuzziness" of its boundary (i.e.  $\varepsilon \sim \frac{\Delta R}{R}$ ). This is identical to the behavior of Heisenberg's partial charge  $\|Q_{R,\Delta R}\Omega\|$  acting on the vacuum. This is result which one can rigorously derive in terms of appropriate test function smearing from the two-point function of a conserved current [23]. This plain area law also follows from 't Hooft's brickwall picture [67].

On the other hand the light sheet picture favours a logarithmic modification of the area law for  $n > 2$ . The picture is that of a volume factor of a "box" where two transverse spatial direction are responsible for an area factor and the third lightlike extension contributes a  $\ln(\frac{R}{\Delta R})$  factor which substitutes the factor from the third spatial extension [20][68]. It is most close to the idea of an inverse Unruh effect since the space box of heat bath statistical mechanics is replaced by a box for which one direction is lightlike (being responsible for the log modification<sup>35</sup>). The abstract nature of the split property and the difficulties in extracting a concrete computation from it (the approximating Hamiltonian is not known) make it presently impossible to decide between a plain area law and its logarithmic modification. This relation between the sharpness  $\varepsilon$  of the localization boundary and the localization entropy/energy replaces the uncertainty relation of QM whereas Heisenberg's uncertainty relation becomes meaningless since there is no position operator in QFT.

The connection between modular localization and thermal aspects may be little known, but there is one system which made it into science fiction and is part of the particle physics folklore: the aforementioned Unruh Gedankenexperiment, which is nearly as old as the close related Hawking effect. In both cases quantum fields become localized, in the first case behind the observer-dependent Rindler

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<sup>35</sup>In [23] it was called the *weak Unruh inverse*; the terminology *strong Unruh inverse* being reserved for an *isomorphism* between a heat bath- and a localization caused- thermal system.

wedge which is the causal shadow region of its horizon (i.e. the wedge itself), and in the second case the less "fleeting" (less observer dependent) situation of the *event horizon* of the Schwarzschild metric. In the first case the important question, which Unruh answered by the construction of a Gedankenexperiment, was *what is the physical meaning of being localized in a wedge  $W$  of Minkowski spacetime?* In this case the modular Hamiltonian is the generator of the  $W$ -preserving Lorentz boost i.e. the  $W$ -localized observable (counter, observer) must be uniformly accelerated in order not to trespass the horizon of the wedge; for him the inertial frame Hamiltonian of Minkowski spacetime is irrelevant, his Hamiltonian is the spectral-symmetric boost generator (instead of the one sided spectrum of the time translation Hamiltonian). This requires to pump energy to accelerate an observable i.e. the Unruh effect is not an "perpetuum mobile" for creating heat, and the vacuum on the global algebra of all operators in Minkowski spacetime keeps its ground state properties with respect to the Poincaré group.

In order to remove one more mystery from the connection of localization with the thermal aspects of the reduced vacuum and the concomitant effect of vacuum polarization, we will now show how the important *crossing property* of particle physics has its explanation in the KMS property of the wedge-restricted vacuum. Although the mystery which surrounded the crossing property for many decades and led to incorrect interpretations (see previous section) will be lifted, some surprise about its unexpected true nature remains; this is what the line added to the title of this section refers to.

For this purpose one starts from the modular operator theory applied the wedge algebra  $\mathcal{A}(W)$  which denotes the operator algebra formally generated by smeared fields whose smearing function support is contained in  $supp f \subset W$ . For this algebra acting on the vacuum  $\Omega$  the Tomita  $S$ -operator is well-known to have the following definition and lead to important operators under polar decomposition[25]

$$\begin{aligned} S_W A \Omega &= A^* \Omega, \quad A \in \mathcal{A}(W), \quad S_W = J_W \Delta_W^{\frac{1}{2}} \\ \Delta_W^{i\tau} &= U(\Lambda(-2\pi\tau)), \quad J_W = S_{scat} J_{W,0}, \quad S_{W,0} = J_{W,0} \Delta_W^{\frac{1}{2}} \end{aligned} \quad (13)$$

Here the modular unitary  $\Delta_W^{it}$  is shared between the interacting and free (incoming) system which carries the additional subscript 0. The antiunitary  $J_{W,0}$  which appear in the polar decomposition of  $S_{W,0}$  represents the reflection on the edge of the wedge  $W$  (TCP, apart from a  $\pi$ -rotation along the wedge) without interaction whereas  $J_W$  includes the interaction in form of the  $S$ -matrix  $S_{scat}$ , which now plays an additional role to the one in scattering theory, namely that of a *modular invariant* between the free (incoming) and the interacting wedge algebra [5].

The equality of the dense domains of the interacting  $S$  with that of the free  $S_0$  i.e.  $dom S = dom S_0 = dom \Delta^{\frac{1}{2}}$  implies that there is a dense set of states, namely those in  $dom \Delta^{\frac{1}{2}}$  which can be generated both in the interaction free algebra  $\mathcal{A}_{in}(W)$  generated by the  $W$ -smeared incoming fields and operators

from the interacting algebra  $\mathcal{A}(W)$ . Hence for each operator  $A \in \mathcal{A}_{in}(W)$  there exists a  $A_{\mathcal{A}(W)} \in \mathcal{A}(W)$  such that

$$A|0\rangle = A_{\mathcal{A}(W)}|0\rangle \quad (14)$$

The definition is well-defined and bijective because the vacuum is separating for both algebras  $\mathcal{A}_{in}(W)$  and  $\mathcal{A}(W)$  and because the dense subspaces  $\mathcal{A}_{in}(W)|0\rangle$  and  $\mathcal{A}(W)|0\rangle$  coincide, both being equal to the domain of the common modular operator  $\Delta^{\frac{1}{2}}$ . We shall refer to the bijective assignment

$$\mathcal{A}_{in}(W) \ni A \rightarrow A_{\mathcal{A}(W)} \in \mathcal{A}(W) \quad (15)$$

as "emulation" (of a interaction-free operator  $A$  in the interacting algebra  $\mathcal{A}(W)$ ). The emulation of a single particle operator was previously called PFG in [32], hence a PFG is the emulat of a smeared free field  $A(f)$  with  $supp f \subset W$  and a Wick-product :  $A(f_1) \dots A(f_n)$  : leads to a multi-particle emulat :  $A(f_1) \dots A(f_n) :_{\mathcal{A}(W)}$

The uniqueness of the emulats is secured by demanding that the dense domain  $\mathcal{A}(W)'|0\rangle$  contains a core of the emulats. From the definition it is clear that emulation is not an algebra homomorphism  $(AB)_{\mathcal{A}(W)} \neq A_{\mathcal{A}(W)}B_{\mathcal{A}(W)}$  and  $(A^*)_{\mathcal{A}(W)} \neq (A_{\mathcal{A}(W)})^*$ . More precisely, it follows that

$$(A_{\mathcal{A}(W)})^*|0\rangle = SA_{\mathcal{A}(W)}|0\rangle = S_{scat}S_0A|0\rangle = S_{scat}A^*|0\rangle = S_{scat}A^*S_{scat}^{-1}|0\rangle \quad (16)$$

With the help of these emulats one can now state the cyclic KMS property for the interacting algebra in a form which is convenient for the later derivation of particle crossing

$$\begin{aligned} \langle 0|BA_{\mathcal{A}(W)}^{(1)}A_{\mathcal{A}(W)}^{(2)}|0\rangle &= \lim_{\tau \rightarrow 1} \langle 0|A_{\mathcal{A}(W)}^{(2)}\Delta^\tau BA_{\mathcal{A}(W)}^{(1)}|0\rangle \\ A^{(1)} &\equiv: A(f_1) \dots A(f_k) :, \quad A^{(2)} \equiv: A(f_{k+1}) \dots A(f_n) :, \quad supp f_i \in W \end{aligned} \quad (17)$$

The analytic content of the crossing relation is that the right hand side is analytic in  $0 < \tau < 1$  and that the analytic continuation of the right hand side from the physical value  $\tau = 0$  to  $\tau = 1$  (1 to  $\Delta$ ) obeys the crossing identity (17). The second line specifies the  $W$ -localized Wick products ( $supp f_i \subset W$ ) which acting on the vacuum convert the right hand side into a formfactor of  $B$  (or formally  $\Delta B$ ) between a  $k$ -particle incoming bra- and a  $n-k$  particle *outgoing* (since the star (16) involves the S-matrix) ket state.

In the absence of interactions i.e. with  $B$  being a Wick composite of a free field, the content of the free KMS relation is a rather direct consequence of the Wick-ordering theorem. The particle content of the identity

$$\begin{aligned} \langle 0|BA_{\mathcal{A}(W)}^{(1)}|\hat{f}_{k+1}, \dots, \hat{f}_n\rangle_{in} &= out \langle \hat{f}_{a,k+1}, \dots, \hat{f}_{a,n}|\Delta B|\hat{f}_1, \dots, \hat{f}_k\rangle_{in} = \\ &= \int \frac{d^3 p_1}{2p_{0,1}} \dots \int \frac{d^3 p_n}{2p_{0,n}} \hat{f}_1(p_1) \dots \hat{f}_n(p_n) out \langle -\bar{p}_{k+1}, \dots, -\bar{p}_n | \Delta^{\frac{1}{2}} B | p_1, \dots, p_k \rangle_{in} \\ out \langle -\bar{p}_{k+1}, \dots, -\bar{p}_n | B | p_1, \dots, p_k \rangle_{in} &:= out \langle \bar{p}_{k+1}, \dots, \bar{p}_n | \Delta^{\frac{1}{2}} B | p_1, \dots, p_k \rangle_{in} \end{aligned} \quad (18)$$



is however highly subtle; the culprit is the uniquely determined but not yet known operator  $A_{\mathcal{A}(W)}^{(1)}$ , which still needs to be computed. The first line uses (16) which converts the action of the conjugate of  $A_{\mathcal{A}(W)}^{(2)}$  on the bra-vacuum into an outgoing particle state vector of antiparticle  $\bar{p}$  in antiparticle wave functions  $\hat{f}_a$ . Their complex conjugate outside the matrixelement is then transformed back into particle wave functions by using  $\Delta^{\frac{1}{2}}$  of the  $\Delta$  for its analytic continuation back to the  $\hat{f}$  (last two lines in (18)). If we could forget the emulation in the middle of the left hand side and equate the resulting integrands in momentum space, we would obtain the "folkloric" version of the crossing identity (which turns out to be only valid for special configurations), but the correct version is much more subtle and interesting.

Let us first be reminded how this problem was resolved for integrable d=1+1 models in [69] for which the plane wave particle states can be uniquely described in terms of rapidities  $\theta$ . The crucial step was to think about analytic transpositions i.e. to consider the vacuum formfactor of two-dimensional models in the rapidity parametrization

$$\langle 0 | B | \theta_1, \theta_2, \dots, \theta_n \rangle_{in} \quad (19)$$

as a locally analytic functions in the  $\theta$ s and that by analytic continuation we can change the order of  $\theta$ s. In that case one arrives at new objects and in order to not confuse these new objects with the trivial result obtained from statistics we, stipulate (using the statistics degeneracy) that for the natural (from left to right decreasing) ordering  $\theta_1 > \dots > \theta_n$  the state in the vacuum formfactor refers to an incoming n-particle state and any other left right ordering denotes a yet unknown object which is defined by analytically continuing starting from the natural order to that obtained by analytic continuation.

In [69] it was shown that for models whose S-matrix is given in terms of an elastic two-particle scattering function  $S^{(2)}(\theta_1 - \theta_2)$  (or a matrix of scattering functions), the analytic transposition of two adjacent  $\theta$  is described by this scattering function. In this case the S-matrix as well as the resulting formfactors are meromorphic functions on the multi- $\theta$  plane. The analytic change of the ordering can be encoded into a wedge localized operator whose positive and negative energy components fulfill the Zamolodchikov-Faddeev algebra relation

$$\begin{aligned} \left( \tilde{A}_{in}(x) \right)_{\mathcal{A}(W)} &= \int_{\partial C} Z^*(\theta) e^{ip(\theta)x} d\theta, \quad C = (0, i\pi) \text{ strip} \\ Z(\theta) Z^*(\theta') &= \delta(\theta - \theta') + S(\theta - \theta' + i\pi) Z(\theta') Z(\theta), \quad Z(\theta) = Z^*(\theta + i\pi) \\ Z^*(\theta) Z^*(\theta') &= S(\theta - \theta') Z^*(\theta') Z^*(\theta) \end{aligned} \quad (20)$$

and the product in the natural order

$$Z^*(\theta_1) \dots Z^*(\theta_n) |0\rangle = |\theta_1, \dots, \theta_n\rangle_{in} \quad (21)$$

$$Z^*(\theta_n) \dots Z^*(\theta_1) |0\rangle = |\theta_1, \dots, \theta_n\rangle_{out} \quad (22)$$

Using the product representation of the n-particle S-matrix in terms of  $S^{(2)}$  it is easy to see that the opposite order corresponds to the outgoing n-particle

state (22). The algebraic encoding of the analytic changes in terms of positioning of noncommutative operators instead of  $\theta$ -ordering in n-particle state vectors is evidently more convenient. In particular one obtains for the action of a  $Z^*(\vartheta)$

on an ordered n-particle state a n+1 particle state multiplied with a numerical factor

$$Z^*(\vartheta) |\theta_1, \dots, \theta_n\rangle_{in} = S_{gs}^{(j)}(\vartheta; \theta_1, \dots, \theta_j) |\theta_1, \dots, \theta_j, \vartheta, \theta_{j+1}, \dots, \theta_n\rangle_{in}, \quad \theta_j < \vartheta < \theta_{j+1} \quad (23)$$

$$S_{gs}^{(j)}(\vartheta; \theta_1, \dots, \theta_j) = S(\vartheta - \theta_1) \dots S(\vartheta - \theta_j) = S^*(\theta_1, \dots, \theta_j) S(\vartheta, \theta_1, \dots, \theta_j) \quad (24)$$

where we call  $S_{gs}^{(j)}(\vartheta; \theta_1, \dots, \theta_j)$  the *grazing shot S-matrix* for  $\vartheta$  impinging on the  $\theta_1, \dots, \theta_n$  cluster<sup>36</sup>. For the generalization to the non-integrable case it is important to express  $S_{gs}$  in terms of a product of ordinary S-matrices (24). For the action of the annihilation operator one obtains a grazing shot S-matrix with  $\vartheta \rightarrow \vartheta + i\pi$  which multiplies an n-1 particle state since once the  $Z(\vartheta)$  has been commuted through the cluster it annihilates the  $\theta_{j+1}$  (the  $\check{\theta}_{j+1}$  indicated that it is missing)

$$Z(\vartheta) |\theta_1, \dots, \theta_n\rangle_{in} = \delta(\vartheta - \theta_{j+1}) S_{gs}^{(j)}(\vartheta + i\pi; \theta_1, \dots, \theta_j) |\theta_1, \dots, \check{\theta}_{j+1}, \dots, \theta_n\rangle_{in} \quad (25)$$

The grazing shot scattering amplitude of a  $\vartheta$ -”bullet” impinging on a  $\theta$ -cluster has a generalization to non-integrable QFT

$$S_{gs}^{(m,n)}(\vartheta, j; \chi, \theta) \equiv \sum_l \int \dots \int d\vartheta_1 \dots d\vartheta_m \langle \chi_1 \dots \chi_m | S^* | \vartheta_1, \dots, \vartheta_l \rangle \cdot \langle \vartheta, \vartheta_1, \dots, \vartheta_l | S | \vartheta, \theta, \theta_1, \dots, \theta_j \rangle \quad (26)$$

We conjecture that this expression describes the analog of (23) i.e.

$${}_{in} \langle \chi_1, \dots, \chi_m | Z^*(\vartheta) | \theta_1, \dots, \theta_n \rangle_{in} = S_{gs}^{(m,n)}(\vartheta, j; \chi, \theta) \quad (27)$$

Note that in this case the re-ordering into the natural order brings in particle state vectors of arbitrary high particle number. The intuitive idea behind the grazing shot S-matrix is that commuting a PFG through a cluster of particles, the result should be trivial if the localization of the wave packet of the particle remains effectively in a large distance from that of the cluster. Such  $Z^\#(\vartheta)$ s cannot be characterized in terms of Z-F like commutation relations.

The argument leading to the action of a one-particle emulator  $Z^\#$  on an incoming n-particle state is the same in both cases. It rests on two assumptions:

1. The only way the interaction enters this action is through the S-matrix

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<sup>36</sup>Note that as a result of momenta conservation the dull S-matrices as well as their grazing shot counterparts for integrable models allow a simpler notation. The matrix elements of the n-particle S-matrix is a combinatorial product of two-particle amplitudes [69].

2. The action is determined with the help of  $\theta$ -reordering defined in terms of commuting the emulat  $Z^\#$  through  $\theta$ -clusters.

The first assumption receives its support from the fact that the interaction enters the modular object  $J = S_{scat}J_{in}$  only through the scattering matrix, hence one expects that the generating emulats of  $\mathcal{A}(W)$  are also determined in terms of  $S_{scat}$  only. The origin of the second requirement is the idea of an analytic change of ordering can be expressed in terms of algebraic properties of emulats acting on particle states. The underlying philosophy is similar to that in [69], one starts from an analytic picture about analytic changes of  $\theta$ -orders in formfactors and converts this with the help of emulation into an algebraic structure whose validity can be directly checked. The crossing property results from the KMS property together with this algebraic structure.

From [32] we know that the operator properties of emulats in the general case are radically different from those for integrable theories. Whereas for the latter the plane wave formulation can be directly justified as a result of the translation invariance of the domains of the emulats (this is the intrinsic definition of "integrable"), the non-integrable emulats only exists as  $Z^\#(f)$  for  $suppf \subset W$ , and their action is only defined on the dense set of W-supported multi-particle states. Hence the above relations have to be smeared with W-localized wave functions; without smearing they should only be understood as relations for bilinear forms as in (27). Their expected bad domain properties should result from the summation over infinitely many intermediate states in the definition of  $S_{gs}^{(n,m)}$ .

What remains to be done is to show that the formal objects defined by the above formulas can be backed up by operators  $A_{in}(f)_{\mathcal{A}(W)}$  (20) with the claimed domain properties and last not least that these operators are wedge dual in the sense

$$\langle \psi | \left[ J A_{in}(\hat{f})_{\mathcal{A}(W)} J, A_{in}(\hat{g})_{\mathcal{A}(W)} \right] | \varphi \rangle = 0, \quad J = S_{scat} J_{in} \quad (28)$$

$$[\mathcal{A}(W'), \mathcal{A}(W)] = 0, \quad \mathcal{A}(W)' = \mathcal{A}(W)' \equiv J \mathcal{A}(W) J \quad (29)$$

which is the wedge duality (29) expressed in terms of the emulats. For integrable models all these checks are straightforward and can be found in [31][28], whereas in the general case they are only reasonably simple in case one of the states is the vacuum  $|\varphi\rangle = |0\rangle$  (in which case the resulting relation can be reduced to wedge duality in the free field algebra). We hope to be able to complete the proofs in a future publication.

With such a difficult task still ahead, it is helpful to recall the aim of this new nonperturbative setting. One objective is to show at least the *existence of models* which in view of being non-integrable can only be *approximated* in a controlled way. In the present setting one would like to see if (in analogy to the integrable case) a crossing symmetric unitary Poincaré invariant S-matrix determines a unique QFT. At least under the two assumptions the answer is unique as it was already known before for integrable models and conjectured

for  $S = 1$ . The existence problem is not answered by referring to the existence of wedge generators but it requires another difficult step namely to show the nontriviality of double cone intersection [28]. Even in the integrable case this second step was anything but simple [28].

There could be a chance that a kind of on-shell perturbation theory may lead to a convergent perturbative construction of emulats together with the S-matrix. One would e.g. start with a lowest order S-matrix in terms of the mass-shell restriction of the interaction polynomial and use it in (28) to start an iteration whose first step is a lowest order emulat. From these data one aims at the next order  $S_{scat}$  and so on. There may be other ways to start an induction with low order emulates. The difference to the Epstein-Glaser iteration for pointlike localized fields is that there are no singular (operator-valued distributions) on-shell operators so that if the singular structure (requiring renormalization) would have been the reason for the divergence of the perturbative series, the on-shell situation in the present case may be better. On shell perturbation theory in terms of the S-matrix has been attempted at the time of the S-matrix bootstrap; from the present setting it is clear that this remained without success since only within a formfactor program one has sufficient structure (the matrix elements if the  $S_{scat}$ -operator represent the formfactor of the identity operator) to start an iteration.

The action of emulates on multiparticle states leads to a rather profound insight into the validity of the crossing identity. It is clear that crossing in the standard form

$$\langle 0 | B | \theta_1, \dots, \theta_k, \theta_{k+1}, \dots, \theta_n \rangle_{in} = {}_{out} \langle \bar{\theta}_{k+1}, \dots, \bar{\theta}_n | U(\Lambda_{W(0,1)}(\pi i)) B | \theta_1, \dots, \theta_k \rangle_{in} \quad (30)$$

$B \in \mathcal{A}(\mathcal{O}), \mathcal{O} \subseteq W_{(0,1)}, \bar{\theta} = \text{antiparticle of } \theta, \theta_1 > \dots > \theta_n$

can only be valid if the particle states are smeared with nonoverlapping wave functions  $\hat{f}$  so that the n-particle state can be expressed in terms of a wave function-smeared naturally ordered product of  $n$   $Z^*$ s applied to the vacuum. In case of overlapping wave functions (the wedge localized wave functions always overlap) we must use the above formulas for the action of emulats in order to disentangle the action of the emulat on the left hand side (18) into particle states; in this process particle states of arbitrary high particle number may enter. In fact without this complication the emulation would be trivial and the KMS relation (17) would be indistinguishable from that of a free field. Even in the *integrable case* the contributions from the interaction modify the LSZ reduction formulas e.g. [69]

$$\begin{aligned} \langle \bar{\theta} | B | \theta_1, \dots, \theta_n \rangle_{in} &= \langle 0 | B | \theta_1, \dots, \theta_n, \theta - i\pi \rangle_{in} + \\ &+ \sum_{j=1}^n \delta(\bar{\theta} - \theta_j) \langle 0 | B | \theta_1, \dots, \hat{\theta}_j, \dots, \theta_n \rangle_{in} S_{gs}^{(j)}(\theta; \theta_1, \dots, \theta_{j-1}) \end{aligned} \quad (31)$$

hence only the  $j=1$  ( $S_{gs}^{(1)} = 1$ ) contact term is equal to what one obtains from the naively derived LSZ reduction formula (which ignores the modifications of

overlapping situations). Only if the outgoing rapidities are "smaller" (smaller  $\theta$ s) than those of incoming  $\theta$  the crossing relation takes the standard form; this is in particular fulfilled if one starts from a vacuum formfactor (30). For higher dimensional QFT the rapidity ordering has to be replaced by the velocity ordering with respect to the wedge region  $W$  [32].

Threshold singularities which lead to a breakdown of the Haag-Ruelle scattering theory and to complicated changes of the LSZ reduction formula are absent when neither the incoming and outgoing wave functions overlap among themselves nor the incoming overlap with the outgoing situation. Hence the crossing relation which connects the vacuum to  $n$ -particle formfactor with the  $k$  to  $n - k$  particle formfactor holds since the ordering requirement can always be fulfilled, but if one starts from a general formfactor there will be complicated threshold modifications if the outgoing configuration is not "smaller" than the incoming.

Hence the important message is that, although the crossing identity is related to the KMS identity of wedge localization, one needs to know the action of the emulators on multiparticle states in order to understand the details of that relation. As in the old days of the (abandoned) bootstrap formalism the *crossing property is part of a constructive setting*. This role goes much beyond that at the time of the dispersion relations. It is part of a new design in which the second quantization functor, which leads from Wigner's representation theoretical approach to interaction-free QFT, is replaced by emulation of free fields (particles) within the wedge-localized interacting operator algebra. In contrast to Lagrangian quantization, which works on a parallelism to classical field theories, the emulation approach is totally intrinsic.

Being the formfactor of the identity operator, the S-matrix plays a special role in this construction.

$$\langle \vartheta_1, \dots, \vartheta_m | S | \theta_1, \dots, \theta_n \rangle = {}_{out} \langle \vartheta_1, \dots, \vartheta_m | \mathbf{1} | \theta_1, \dots, \theta_n \rangle_{in} \quad (32)$$

As a result of the energy-momentum conservation it is not possible for the important case of  $2 \rightarrow 2$  particle scattering to cross a single particle, one rather has to cross a pair, one from the incoming and one from the outgoing configuration. The crossing property for pair crossing for the elastic S-matrix is the only case for which the necessary analytic properties were derived already at the time of the dispersion relations [41]. The authors applied the complicated theory of analytic functions of several variables; this method did not reveal much about the conceptual environment of crossing and according to my best knowledge it was not extended to other cases.

In the integrable case the analytic change leads to a representation of the permutation group i.e. the analytic change does not depend on the path on which it was carried out [28]. This is radically different in the general case; the above change involving the grazing shot S-matrix only holds for the direct path (directly pulling an  $Z^\#$  operator through a cluster without zigzagging). This suggests analogies with other cases in QFT for which operator changes correspond to analytic changes of positions in correlation functions. In a QFT of

Wightman correlation functions with standard spacelike commutation relations the analytically continued correlation functions are uni-valued in the Bargman-Hall-Wightman domain. On the other hand in a  $d=1+2$  QFT with plektonic (braid group) commutation relation [64] the uni-valuedness in the BHW domain breaks down and the analytically continued correlation depends on which analytic *path* the coordinates have been changed. Could there be a similar relation between generating fields of wedge algebras constructed by emulation and their analytic changes in formfactors? The algebraization in terms of emulats suggest that this is the case

The usual form of the crossing relates different formfactors while maintaining the sum of bra +ket particles. It only holds if the wave functions of the ket particles do not overlap those in the bra state. Otherwise the usual form breaks down as a result of threshold singularities. In the general case the analytic change couples the cluster of the crossed particles to *all other particles* subject to the superselection rules. This amounts to an on-shell realization of a radical form of Murphy's law: *all that can be coupled* (not forbidden by superselection rules) *will be coupled*. Integrable theories are precisely those which are protected against Murphy's law.

As in classical and quantum mechanics, an explicit construction of nonintegrable QFT models is impossible. The aim of viewing emulation as a generalization of Wigner's intrinsic representation theoretical approach to the realm of interactions is to bring QFT to its conceptual closure by solving its two remaining fundamental problems: proving existence of models (in particular of physically interesting models) and finding mathematically controllable approximation methods which could replace the standard diverging perturbation series for correlation functions of renormalized fields.

The finiteness of on-shell objects was the main reason behind the various S-matrix projects which arose in the late 50s as an attempt to extend the successful derivation and experimental confirmation of dispersion relations into a full S-matrix theory project. The present modular localization setting shows why such attempts had no chance to succeed. An S-matrix construction without its natural conceptual embedding as a relative modular invariant into an approach based on modular theory of wedge localization does not present enough structure to start computations; unitarity, Poincaré invariance and the crossing property of an S-matrix are too general for providing a computational basis. On the other hand ideas to enrich the S-matrix setting with additional assumptions (as e.g. Mandelstam's postulated spectral representation for scattering amplitudes) had a chance, at least as long as they did not lead to outright contradictions with properties following from the principles of local quantum physics.

As shown in the previous section, later S-matrix projects, as the dual model and its string theory extension really did lead to such contradictions. The present approach shows in a clear form that *the crossing on which Veneziano [65] constructed his dual model has nothing to do with the crossing in the sense of particle physics*. Rather what was called crossing in the dual model referred to a kind of "field crossing" in Mellin transforms of converging global operator expansions in conformal 4-point-functions [19] in which meromorphic functions

arise, which have their first order poles on a "dimensional trajectory" defined by scale dimensions of conformal (composite) fields. It also makes no physical sense to subject Mellin transforms to a unitarization process (interpreting the dual model as lowest order of a new S-matrix construction), since the Hilbert space properties of Mellin transforms and those from (approximations) of unitary S-matrices are totally different.

Modular localization is also at odds with the idea of embedding a lower dimensional QFT into a higher dimensional one. This is only possible in QM, which has no intrinsic localization but only the one related to Born's probability interpretation: a linear chain of oscillators does not "feel" the space into which it is (it is up to the acting physicist where to place it) embedded.

Proposals which are the result of a lack of foundational knowledge about local quantum physics blended with an uncritical (if not to say messianic) use of metaphoric analogies can carry a highly speculative science as particle physics into a dead allay, unless a corrective critique rescues the situation in time. I claim that this (without the rescue) is what exactly happened with string theory and explains why after 5 decades theoreticians stand almost empty-handed in front of LHC; those few individuals who tried to rescue the situation were not listened to (had no status in a trend-driven world of science). As in case of most sociological/political derailments, the wrong path has to play out to the end, before something different can be started.

The present results may be summarized as follows. For integrable models the bijective correspondence of wedge localized interacting operators leads to a solution of the existence problem, so that the previously calculated formfactors are really those of existing QFTs. In that case a description of these operators as deformed free fields is very helpful [66]. Their proximity to free fields reflects itself in the fact that in  $d=1+1$  (and only there) purely elastic S-matrix cannot be distinguished from an interaction-free situation by cluster factorization. For non-integrable models one only has a consistent constructive idea, the check of its correctness as well as its usefulness requires more investigations.

In spite of the critical remarks about pure S-matrix constructions, *Mandelstam's step to place the S-matrix into a computational approach was very important*, even if his later closing of ranks with dual model ideas ended in a blind allay. Taking the original idea of using the S-matrix already at the start of computations and combining it with Haag's idea of local quantum physics (which led to modular localization), one obtains a powerful new setting for a rigorously controlled particle theory project.

### 3.3 Résumé and concluding remarks

The success of the dispersion relation project led to the first nonperturbative S-matrix based setting of particle physics. In form of the S-matrix bootstrap it started from correct "axioms", its weakness was the lack of an operational way of their implementation. Concepts as "maximal analyticity" were conceptually as well as mathematically too vague and the S-matrix bootstrap setting remained without concrete results.

The more constructive aftermath of the dispersion relation era started with Mandelstam's conjectured two-variable representation, and moved via the dual model to the canonically quantized Nambu-Goto Lagrangians and its  $d=1+9$  dimensional supersymmetric infinite component QFT called "superstring". It was more concrete and led to many detailed calculations but, as was shown in the present work, *it erred on the nature of the crossing property in particle physics* and more generally got confused on the subtle issue of quantum meaning of localization. Although QFT was born in the aftermath of the Einstein-Jordan conundrum of fluctuation in a subvolume, whose solution required the deep (and at that time not available) notion of modular localization for its complete solution [61] [7], its age-old incomplete understanding did not impede progress in perturbative aspects QFT. The obvious reason is that renormalized perturbation theory is a "self-seller" since, even in its most conceptual-mathematical form in terms of the Epstein-Glaser approach, it can be implemented without knowing the deeper intrinsic *modular localization* aspects of QFT.

The return to a problem. which was for a very long time (since the very discovery of QFT by Jordan) looming in the background outside of the conceptual range, can be expected to lead to many other new perspectives in particle theory, which could even bring about a different view about quantization and perturbation theory. The motto of a paper written at the turn of the century by Borchers [71] with the title: *On revolutionizing quantum field theory with Tomita's modular theory* is now slowly becoming reality.

Even such a somewhat stagnating subject as gauge theory may get into gears again. On the foundational level gauge theory (and any other QFT involving massless higher spin potentials) from the point of view of Wigner representation theory presents a *deep clash of the Hilbert space structure and modular localization*. The ( $m=0$ ,  $s=1$ ) Wigner representation theory only contains (upon covariantization) generating pointlike  $F_{\mu\nu}(x)$  wave functions but there are no pointlike  $A_\mu(x)$  which (as wave function-valued distributions) generate the Wigner wave function space. But as everybody was forced to accept, one cannot formulate interactions without potentials i.e. only in terms of pointlike couplings of quantum matter with field strengths. The quantization approach follows the Lagrangian quantization *parallelism to classical theory*, except that this clash has no analog in the classical theory (i.e.  $A_\mu^{class}(x)$  exists as any other classical field). This leaves two possibilities in order to arrive at a QFT: either quantize and loose the Hilbert space (calculate instead in an indefinite space and return to a Hilbert space with the help of e.g. the BRST formalism), or use covariant semiinfinite string-localized vector potentials  $A_\mu(x)$  in Wigner space and try to introduce those objects into interaction densities by extending the locality-based Epstein-Glaser setting.

Whereas the first approach does not contain any physical (off-shell) charged matter fields, and one is forced to turn to momentum space recipes for photon-inclusive cross sections for scattering of charged particles thus sidelining a space-time (off shell) understanding (as that involved in the LSZ reduction formula) of charge-carrying objects in favour of a prescription for momentum space photon-inclusive cross sections, the string-localized approach in a Hilbert space opens



other possibilities. One now has the chance to understand the nonlocal quantum aspects of charged matter, which permits at best a string-localized generating charged matter field, in terms of the lowest order interaction.

This explains all those properties of charge carriers whose existence has been previously inferred on the basis of structural arguments based on the quantum Gauss law [25] (which are in fact known to be in contradiction with the formal aspects of the perturbative ghost formalism [9][10]). The standard gauge approach only admits a charge neutral BRST-invariant Hilbert space and one loses the chance to obtain a deeper understanding of spacetime properties of charge carriers; even some charge neutral objects (e.g. as two localized opposite charges with a gauge bridge between them) are not part of the BRST formalism. The mechanism which produces string-localized charge-carrying generating fields remains obscure since the interaction density just looks like that of any other model which couples pointlike fields and leads to pointlike interacting fields; the reason why coupling of zero mass  $s < 1$  fields do not lead to infrared problems whereas couplings of  $s > 1$  do remains unexplained.

The formalism based on string-localized potentials explains the string-localization of charge carrying objects required by the quantum Gauss law as a (perturbative) transfer of string localization from potentials to the matter fields a process in which the potential gets away with still keeping its associated pointlike field strength, the string localization of the charge carrier cannot be undone by any linear operation.

The reformulation of gauge theory in terms of string-localization also leads to a better understanding of the Schwinger-Higgs screening mechanism [16] which was known to experts in the 70s but this nice resresentation of the Higgs phenomenon was lost in the maelstrom of time, leaving behind the today's form of the Higgs model with its sometimes misunderstood idea of a symmetry breaking (which symmetry? Gauge is not a physical symmetry). We refer for the description of the original Higgs model as arising from Schwinger-Higgs charged screening of scalar QED as well as some speculative remarks about a possibility to have a Higgs free massive YM model<sup>37</sup> in the string-localized setting to the existing literature [10]. The string-localized version is the only formulation in which massive potentials pass smoothly to their massless counterparts.

Another lost concept whose recollection could have changed the direction of the dispute about the mathematical AdS-CFT correspondence (and prevented a tsunami of thousands of publications on such a narrow subject without any tangible result) is the Haag-Swieca-Buchholz result [25] about phase space degrees of freedom in QFT (mentioned in section 4). It plays an important role once one goes beyond Lagrangian quantization, but still wants to keep as much of its physical properties in a more intrinsic "axiomatic" setting. With its deep (and not completely understood) relation to the causal shadow property and the existence of global thermodynamic equilibrium states [25], it is one of the

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<sup>37</sup>Even if the existence of the Higgs particle (i.e. the particle associated with the real scalar field which survives after the charge of the complex scalar QED has been screened) will be observed, it is always better to have theoretical alternatives since less desperation enhances the credibility of results.

pillars of Local Quantum Physics. Together with modular localization these results are expected to become increasingly important in more intrinsic settings of QFT beyond perturbation theory.

The progress on such issues led to a foundational understanding of integrable models [28][43] [47] and exposes for the first time the limitations of the parallelism to classical physics known as (Lagrangian, canonical) quantization which extend in both directions: for most integrable models no Lagrangian is known (and not needed for their construction) and there are families of mathematically well-defined classical Lagrangians which have no counterpart in QFT<sup>38</sup>. The critique of string theory in section 4 is closely related to such insights.

At this point a disquieting question comes to one's mind: have physicists working in present day particle theory become less capable or has the subject reached a dead end? It is my firm conviction that neither is true. What happened is that the great progress in the first decades after worldwar II was achieved with a relative simple theoretical investments which were sufficient to discover important results, but were not quite adequate for their foundational safeguarding within QFT, which would have been necessary to obtain a strong conceptual mathematical platform for further innovative explorations. As Feynman once expressed, innovative conquests often require to bounce into the "blue yonder" but since many of such attempts end in failure one needs a strong base to which one may safely return. The situation gets completely out of hand if there is no such secure base and even reputable and charismatic leaders of the meanwhile globalized scientific communities loose their critical brakes. The result is a conceptual trap from which there is no escape; even mathematical consistency may in the absence of a physical conceptual guide lead to bizarre "results" [72]. In this way the vernacular "many people cannot err" is turned into its opposite.

I have criticized the various pure S-matrix projects; but I tried to be careful not to permit the inglorious end of the dual model/string theory to invalidate the importance of Mandelstam's S-matrix based project which started in the aftermath of the successful project of dispersion relations and the LSZ scattering theory. By emphasizing the importance of the crossing property and proposing a spectral representation for its exploration, he initiated a fresh start which went beyond older failed attempts.

This was not always my belief, in fact as an adherent of LQP (local quantum physics) I considered the S-matrix as an object which belongs to the roof and not to the foundation of a QFT construction. There is some irony that, under the influence of the bootstrap-formfactor program for factorizing models, I had to change my view and take note that in the form of a *relative modular invariant of wedge localization* the S-matrix becomes part of the constructive foundations. This point of view has meanwhile become the credo of a small but growing community [73] of highly dedicated young researchers. In fact, I we

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<sup>38</sup>Any Lagrangian in a  $d > 2$  with fields whose index space is noncompact does not have a quantum counterpart unless the indices are tensor/spinor indices associated with the Minkowski spacetime on which the field lives. Compact index spaces are allowed and represent inner symmetries [43].

argued above, it is this completely new view about the foundational aspects of localization in QFT which sheds new light on LHC relevant problems (as the Higgs issue) and renews the interest in old problems which had already been forgotten in the maelstrom of (preelectronic) time.

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